Basics of input-output analysis: theory and empirics

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Historical preliminaries

- Primitive pronouncements of IOA in early civilizations:
  - Mesopotamia (modern-day Iraq) some four millennia ago:

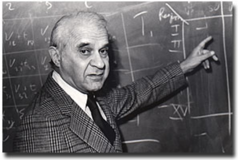
> *"The main entrance to its capital, Babylon, the Ishtar gate, is known to have displayed every year a summary account of the economy's production performance in terms of units of grain, specifically barley. The information given on clay tablets was total output of barley during the year, the input of barley needed directly and indirectly to produce that output, and the surplus product, that is the difference between the output and the input of grain. The input comprised the amount of barley needed for seed and subsistence of workers, overseers and animals plus the barley equivalent of other indispensable inputs. This was a simple scheme of national accounting and it served a useful task. It informed the rulers and (literate) population alike about the amount of barley available for other than reproductive purposes. In years with good harvests this amount was huge; in years with bad harvests it was small; in exceptionally bad years it was negative. Over a succession of good and bad harvests the summary account indicated the average surplus-creating capacity of the economy and thus its capability to cater for other social needs, to engage in wars and conquests, etc."* (Kurz 2011, p. 29)

  - Simple IOA centered on a corn model to estimate population and area
    - The state of Ebla, middle of the third millennium BC (Renger 1986)

Historical preliminaries

- Political economy arising as an independent discipline:
  - Sir William Petty (1623 - 1687): Political Arithmetick
  - Richard Cantillon (1680s - 1734): Essai
  - The Mercantilists: 17th and 18th centuries
- François Quesnay (1694 - 1774) and the Physiocrats
  - Tableau Économique (link):
    Flows of production/cash between landlords (proprietary class), farmers (productive class) and artisans and merchants (sterile class)
- Mathematical formalization
  - Achille-Nicolas Isnard (1748 - 1803): circular flow of income and expenditure as a system of equations
  - Robert Torrens (1780 - 1864)
  - Karl Marx (1818 – 1883)
  - Léon Walras (1834 - 1910)
  - Piero Sraffa (1898 - 1983)
  - Vladimir Karpovich Dmitriev (1868 - 1913)
  - Ladislaus von Bortkiewicz (1868 - 1931)

For further details, see e.g. Kurz and Salvadori (2000), Kurz (2011)

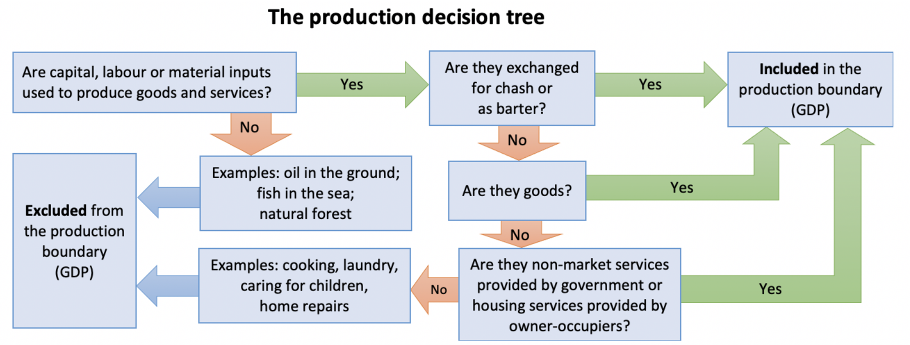


- Wassily W. Leontief (1906 - 1999): founding father of IO economics
  - Leontief, 1928: "The economy as a circular flow" (in German), partial exposition of an (hypothetical) IO system
  - Leontief, 1936: "Quantitative input-output relations in the economic system of the United States", RES, Vol. 18, pp. 105-125.
- He was awarded The Sveriges Riksbank Prize in Economic Sciences in Memory of Alfred Nobel in 1973
  "for the development of the input-output method and for its application to important economic problems"
- Contribution: Creator of the input-output technique, a method that provides tools for a systematic analysis of the complicated inter-industry transactions in an economy (Source: Nobelprize.org).

Dietzenbacher and Lahr (2004), *Wassily Leontief and Input-Output Economics*, Cambridge University Press: Cambridge

"Today, the field of input-output analysis embraces any study that uses data in the format of (or somehow related to) input-output tables, employs an input-output technique as an analytical tool, or develops techniques for producing input-output accounts. The input-output literature covers material across a wide range of: theoretical backgrounds (e.g. classical, neoclassical, Walrasian, Keynesian, Ricardian, Marxian, and Sraffian economics); topics (e.g. growth, welfare, interdependence, (dis)equilibrium, and prices); policy issues (e.g. income distribution, employment, investments, migration, energy consumption, and environment); analytical frameworks (e.g. static, comparative static, dynamic, structural, spatial, and open versus closed); units and levels of analysis (e.g. enterprises, industries, metropolitan areas, regions, multiple regions, single nations, groups of countries, and the world); objects of analysis (e.g. goods and services, materials in physical quantities, prices, innovations, patented inventions, citations, information, and people); and technical focuses (e.g. data collection and compilation of input-output tables, economic theory, and applied mathematics)." (pp. xix-xx)

What output includes and excludes



Source: Adopted from Lequiller and Blades (2014, p. 108)

Output is (generally) valued at **basic prices** corresponding to the revenue per unit of products sold that remain in the hands of the producer, which thus exclude taxes on products but include subsidies received on products

Taxes on production and imports

- Taxes on products (payable *per unit* of some good or service)
  - Value added tax (VAT)
  - Taxes and duties on imports, excluding VAT
    - Import duties (payable according to *customs tariff schedules*)
    - Taxes on imports, excluding VAT and duties
      - Excise duties on e.g. alcoholic beverages, tobacco and fuels
      - General sales taxes
      - Taxes on specific services provided by non-resident enterprises
      - Profits of public enterprises with import monopoly; Etc.
  - Taxes on products, except VAT and import taxes
    - Excise duties and consumption taxes
    - General sales or turnover taxes
    - Taxes on financial and capital transactions
    - Taxes on specific services (transportation, insurance, entertainment, hotels, etc.)
    - Export duties; Etc
- Other taxes on production (*regardless* of production profitability)
  - Recurrent taxes on land, buildings or other structures
  - Taxes on the use of fixed assets (e.g. vehicles, ships, aircraft, machinery or equipment)
  - Taxes on payroll or work force
  - Business and professional licences
  - Taxes on pollution; Etc

Valuation concepts in the SNAs

- Basic price measures the amount retained by the producer
- Producer's price is the price, excluding VAT, that the producer invoices to the purchaser
- Purchaser's price is the price the purchaser pays for the products

Basic prices

+ Taxes on products, excluding invoiced VAT

- Subsidies on products

= **Producers' prices**

+ VAT not deductible by the purchaser

+ Transport charges separately invoiced

+ Wholesalers' and retailers' margins

= **Purchasers' prices**

Intermediate vs. final uses/expenditures

- **Intermediate uses** consist of goods and services that are consumed (used-up or transformed) as inputs in a production process within the economic territory; **final uses** comprises all other goods and services.
  - Independent of the product nature:
    – Potatoes bought by households are “final”, but the same potatoes bought by restaurants are “intermediate”
    – Steel sheet is often an “intermediate”, but becomes “final” when stocked for future consumption, or is exported
  - “Final” therefore simply refers to all the goods and services used during the period that are not entirely consumed (used-up or transformed) in a production process in the course of that same accounting period (Lequiller and Blades 2014)
- Intermediate consumption excludes the *labour of the internal workforce* and the *services provided by plant and machinery, offices and factory buildings*. Consumption of fixed assets is recorded as consumption of fixed capital

Final uses/expenditures

- Households' final consumption expenditure
  - Purchases of goods/services used by HHs to meet their everyday needs
  - Services of owner-occupied dwellings (imputed rents)
  - Own-account consumption goods
  - Income in kind
  - Financial intermediation services indirectly measured (FISIM)
- Final consumption expenditure by general government
  - Individual consumption expenditure: public education, public healthcare, etc.
  - Collective consumption expenditure: safety and order, defence, home affairs, economic affairs, environmental protection, ministries, etc.
- Final consumption expenditure of the NPISHs
  - NPISH: political parties, trade unions, religious organisations, sports clubs, cultural associations, charities, etc.
  - As non-market producer, treated similarly as the general government

Final uses/expenditures (contd.)

- Gross fixed capital formation (GFCF)
  - Net acquisition of produced fixed assets - assets intended for use in the production of other goods/services for a period of more than one year
    - Material (or tangible) fixed assets:
      - Dwellings (excluding land)
      - Other buildings and structures (incl. major improvements to land)
      - Machinery and equipment (e.g. ships, cars and computers)
      - Weapons systems
      - Cultivated biological resources (e.g. trees and livestock)
      - *Costs of ownership transfer on *non-produced* assets, like land, contracts, leases and licences (e.g. transportation and (dis)installation costs, professional charges, taxes, terminal costs)
    - Intangible fixed assets (or intellectual property products, IPPs):
      - R&D, including the production of freely available R&D
      - Mineral exploration and evaluation (spending on the search for oil or mineral deposits)
      - Computer software (standard or developed in-house, originals or copies of originals) and databases
      - Entertainment, literary or artistic originals (e.g. films, novels or music)
      - Other intellectual property rights

Final uses/expenditures (contd.)

- GFCF (contd.)
  - Net acquisitions = Purchases of fixed assets - Sales of fixed assets on the second-hand market
- Changes in inventories
  - Materials and supplies: stocks of inputs to be used later as intermediate consumption; strategic stocks (food, oil, stocks for intervention on agricultural markets) managed by the government
  - Work-in-progress: output produced that is not yet finished
  - Finished goods: stocks of finished goods that have not yet been sold
  - Goods for resale: stocks of goods purchased for resale, found mainly in wholesale and retail distribution
- Net acquisitions of valuables
  - Bought and held primarily as stores of value
  - Precious stones and metals (e.g. diamonds, non-monetary gold, platinum, silver)
  - Antiques and other art objects (e.g. paintings, sculptures)
  - Other valuables (e.g. jewellery fashioned out of precious stones and metals, collectors' items)

Final uses/expenditures (contd.)

- Exports and imports of goods and services
  - Transactions in goods and services from/to residents to/from non-residents
  - Recorded on the basis of change in (economic) ownership between residents and non-residents
    - Goods for processing: fee paid to a non-resident processing enterprise is recorded as an import of services
    - Merchanting: purchase of a good by a resident from a non-resident and the subsequent resale of the good to another non-resident, without the good entering the merchant's economy
  - *Detailed* figures for imports of goods are valued at the cost-insurance-freight (CIF) price at the border of the importing country
    - CIF price is the price of a good delivered at the frontier of the importing country, or the price of a service delivered to a resident, *before* the payment of any import duties or other taxes on imports or trade and transport margins within the country
  - Exports are valued at the free on board (FOB) price at the border of the exporting country: includes transport and distributive services up to that point of the border (incl. cost of loading onto a carrier) and taxes less subsidies on the goods exported (but *not* the transport and insurance costs further to the importing country's frontier)

Gross and net value added

- Gross value added (GVA) at basic prices = Output at basic prices – Intermediate consumption at purchasers' prices
  - GVA could be thought of as "the amount of money generated by production that remains available to pay:
    - wages and salaries and social contributions (compensation of employees);
    - production taxes (other than that on products) net of operating subsidies;
    - replacement of equipment gradually worn out during production (consumption of fixed capital);
    - interest payments on loans;
    - dividends paid to shareholders;
    - purchase of new equipment; and
    - financial saving – or the firms' investment in financial products" (Lequiller and Blades 2014, p. 113)
  - GVA represents the contribution of labour and capital to the production process
- GVA at factor cost = GVA at basic prices - Other taxes less subsidies on production
- Net value added (NVA) = GVA - Consumption of fixed capital

Three approaches to GDP measurement

GDP at market prices can be measured using any of the following approaches:

1. **Production approach**:
   Gross Value Added (GVA) at bp + Taxes less subsidies on products
2. **Income approach**:
   Compensation of employees + Gross operating surplus + Other taxes less subsidies on production + Taxes less subsidies on products
3. **Expenditure approach**:
   Private final consumption expenditure at pp + Government final consumption expenditure at pp + Gross capital formation at pp + Exports at FOB - Imports at CIF

Exercise: Check these methods using using 2016 Spanish IOT

Closer look at the IO interrelations

"Input-output analysis is a method of systematically quantifying the mutual interrelationships among the various sectors of a complex economic system. In practical terms, the economic system to which it is applied may be as large as a nation or even the entire world economy, or as small as the economy of a metropolitan area or even a single enterprise." (Leontief, 1985, Input-Output Analysis, in *Encyclopedia of Materials Science and Engineering*, Oxford: Pergamon Press)

Simplified input-output table expressed in value terms

from / *into*	Sector 1: Agriculture ($)	Sector 2: Manufacture ($)	Sector 3: Households ($)	Total Output ($)
Sector 1: Agriculture	50	40	110	200
Sector 2: Manufacture	70	30	150	250
Sector 3: Households	80	180	40	300
Total Input ($)	220	250	300	

Source: Leontief (1985, Table 2.2)

Table 15.4: Input-output table of domestic output at basic prices (Version A)

Millions of Euro

No.		INPUT OF PRODUCTION ACTIVITIES						FINAL USES					Output at basic prices
		Agri-cul-ture	Manufac-turing	Con-struc-tion	Trade	Busi-ness services	Other services	Private con-sumption	Govern-ment con-sump-tion	Gross fixed capital forma-tion	Chang-es in inven-tories	Exports	
	PRODUCTS (CPA)	1	2	3	4	5	6	7	8	9	10	11	12
1	Agriculture	1 131	25 480	1	607	710	762	8 500	16	2 975	- 6	3 734	43 910
2	Manufacturing	7 930	304 584	64 167	41 082	11 981	30 360	197 792	8 588	91 692	7 559	313 711	1 079 446
3	Construction	426	7 334	3 875	5 296	23 457	9 155	3 457	742	191 715		149	245 606
4	Trade	3 559	72 717	14 190	74 399	10 835	21 008	269 663	13 492	14 155		46 045	540 063
5	Business services	3 637	96 115	31 027	65 755	193 176	34 223	214 757	10 061	30 124		13 612	692 487
6	Other services	1 552	14 986	1 747	11 225	15 058	22 070	119 504	317 251	3 483		2 042	508 918
7	Domestic products	18 235	521 216	115 007	198 364	255 217	117 578	813 673	350 150	334 144	7 553	379 293	3 110 430
8	Imported products	2 927	156 703	13 427	21 943	13 371	13 772	80 187	2 970	41 436	- 4 233	42 597	385 100
9	Taxes less subsidies on products	1 084	6 505	1 548	8 349	8 473	12 551	107 200	3 670	28 660	260	- 1 160	177 140
10	Intermediate consumption/Final use at purchasers' prices	22 246	684 424	129 982	228 656	277 061	143 901	1 001 060	356 790	404 240	3 580	420 730	3 672 670
11	Compensation of employees	9 382	296 464	78 819	214 450	124 810	272 975						996 900
12	Other net taxes on production	- 2 012	1 457	963	2 748	5 946	- 8 602						500
13	Consumption of fixed capital	7 871	63 769	5 860	41 100	98 610	49 260						266 470
14	Net operating surplus	6 423	33 332	29 982	53 109	186 060	51 384						360 290
15	Value added at basic prices	21 664	395 022	115 624	311 407	415 426	365 017						1 624 160
16	Output at basic prices	43 910	1 079 446	245 606	540 063	692 487	508 918	1 001 060	356 790	404 240	3 580	420 730	-

Germany 1995

Source: European Commission (2008), *Eurostat Manual of Supply, Use and Input-Output Tables* (p. 483) notation

Input-output table (IOT) in value terms

	Intermediate demand of industries	Local final demand and exports	Totals
Industries' sales	$\boldsymbol{Z}$	$\boldsymbol{f}$	$\boldsymbol{x}$
Imports	$\boldsymbol{m}'$	m_f	m_{tot}
GVA (incl. TLS)	$\boldsymbol{v}'$	v_f	v_{tot}
Totals	$\boldsymbol{x}'$	f_{tot}	

- Notice: $f_{tot} = m_{tot} + v_{tot} \quad \Rightarrow \quad f_{tot} - m_{tot} = v_{tot}$. Interpretation?
- **Output-side accounting identity** per industry:
  Total output = Intermediate demand + Final demand
- In matrix notation: $\boldsymbol{x} = \boldsymbol{Z}\boldsymbol{\imath} + \boldsymbol{f}$, where $\boldsymbol{\imath}' = (1, 1, \cdots, 1)$.
- Define domestic input coefficients ("cooking recipes", "production recipes") as:

$$a_{ij} = \frac{z_{ij}}{x_j} \quad \text{for all } i, j = 1, 2, \ldots, n. \tag{1}$$

Interpretation of a_{ij}: euro's amount of *domestic* industry i's output needed per euro's worth of industry j's total output.

closed IO

- Equation (1) is equivalent to:

$$z_{ij} = a_{ij}x_j \quad \text{or} \quad \boldsymbol{Z} = \boldsymbol{A}\hat{\boldsymbol{x}}$$

- Plugging this in the output-side accounting identity yields:

$$\begin{aligned} \boldsymbol{x} &= \boldsymbol{Z}\boldsymbol{\imath} + \boldsymbol{f} \\ &= \boldsymbol{A}\hat{\boldsymbol{x}}\boldsymbol{\imath} + \boldsymbol{f} \\ &= \boldsymbol{A}\boldsymbol{x} + \boldsymbol{f} \end{aligned}$$

- Quantity IO *model*: Assuming that $\boldsymbol{A}$ is fixed, find $\boldsymbol{x}$ for an exogeniously specified $\boldsymbol{f}$
- Hence, we arrive at a system of n linear equations:

$$(\boldsymbol{I} - \boldsymbol{A})\boldsymbol{x} = \boldsymbol{f} \tag{2}$$

Thus, one can readily use the techniques of Linear Algebra!

Leontief inverse

The inverse of $(\boldsymbol{I} - \boldsymbol{A})$ is known as the **Leontief inverse** or the **total requirements matrix** and has the following two equivalent formulations:

$$\boldsymbol{L} = (\boldsymbol{I} - \boldsymbol{A})^{-1} \tag{3}$$

$$\boldsymbol{L} = \boldsymbol{I} + \boldsymbol{A} + \boldsymbol{A}^2 + \boldsymbol{A}^3 + \cdots \tag{4}$$

- Equations (2) and (3) together imply the following fundamental quantity equation in IOA:

$$\boldsymbol{x} = \boldsymbol{L}\boldsymbol{f} \tag{5}$$

- Interpretation of entries in $\boldsymbol{L}$: take $\boldsymbol{f} = \boldsymbol{e}_j$, where $\boldsymbol{e}_j$ is the j-th column of $\boldsymbol{I}$. For example, take $j = 1$:

$$\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} l_{11} & l_{12} & \dots & l_{1n} \\ l_{21} & l_{22} & \dots & l_{2n} \\ \vdots & \vdots & & \vdots \\ l_{n1} & l_{n2} & \dots & l_{nn} \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix} = \begin{bmatrix} l_{11} \\ l_{21} \\ \vdots \\ l_{n1} \end{bmatrix}.$$

INTERPRETATION OF l_{ij}

Physical IOT: l_{ij} gives the (extra) output in industry i that is directly and indirectly required to satisfy one (extra) unit of final demand in industry j.
Monetary IOT: l_{ij} gives the (extra) output in euros in industry i that is required to satisfy one (extra) euro of final demand in industry j. With fixed prices, all changes are caused by simultaneous quantity increases.

Round-by-round effects from the power series (4):

- Consider an arbitrary *changes* in final demands, $\Delta \boldsymbol{f}$:

$$\begin{aligned} \Delta \boldsymbol{x} = \Delta \boldsymbol{f} \quad & \text{Round 0: initial demands} \\ + \boldsymbol{A}\Delta \boldsymbol{f} \quad & \text{Round 1: direct inputs requirements} \\ + \boldsymbol{A}(\boldsymbol{A}\Delta \boldsymbol{f}) \quad & \text{Round 2: indirect inputs requirements} \\ + \boldsymbol{A}(\boldsymbol{A}\boldsymbol{A}\Delta \boldsymbol{f}) \quad & \text{Round 3: indirect inputs requirements} \\ \vdots \quad & \end{aligned}$$

- If $\Delta \boldsymbol{f} = \boldsymbol{e}_j$ and $i \neq j$, then:
  $\Delta x_i = 0 + a_{ij} + \sum_{k=1}^{n} a_{ik} a_{kj} + \sum_{k=1}^{n} \sum_{h=1}^{n} a_{ik} a_{kh} a_{hj} + \cdots$.
  Question: How the above expression changes for $i = j$?

Closing the model with respect to households (HHs)

- HHs earn income in exchange for their labor inputs to production
- As consumers, HHs spend their income on products in "rather well patterned ways" (Miller and Blair 2009, p. 35)

Visualising IOT with endogenous aggregate HHs

	Intermediate demand of industries	Households consumption	Other final uses	Totals
Industries sales	$\boldsymbol{Z}$	$\boldsymbol{h}$	$\boldsymbol{f}^*$	$\boldsymbol{x}$
Households labor	$\boldsymbol{w}'$	0	0	w_{tot}
Total imports	$\boldsymbol{m}'$	m_h	m_{f^*}	m_{tot}
Other GVA	$(\boldsymbol{v}^*)'$	v_h	v_{f^*}	v^*_{tot}
Totals	$\boldsymbol{x}'$	h_{tot}	f^*_{tot}	

- Singling out HHs consumption (labor) from final demand (value added):
  $\boldsymbol{f} = \boldsymbol{h} + \boldsymbol{f}^*$, where $\boldsymbol{f}^*$ is final demand excluding HHs consumption $\boldsymbol{h}$
  $\boldsymbol{v} = \boldsymbol{w} + \boldsymbol{v}^*$, where $\boldsymbol{v}^*$ is value added excluding labor compensation $\boldsymbol{w}$

open IO Miyazawa

Endogenizing aggregate households

- Treat HHs similar to industries in the open IO model:
  $\boldsymbol{h}_c \equiv$ "consumption coefficients" vector: $h_{ic} = h_i / w_{tot}$ where w_{tot} is the total output - measured by **income earned** - of the HHs sector
  $\boldsymbol{w}_c \equiv$ "HHs input coefficients" vector: $w_{cj} = w_j / x_j$ for all $j = 1, \ldots, n$
- The output-side accounting identity of the expanded IO system is:

$$\boldsymbol{x} = \boldsymbol{Z}\boldsymbol{\imath} + \boldsymbol{h} + \boldsymbol{f}^*$$
$$w_{tot} = \boldsymbol{w}'\boldsymbol{\imath}$$

- The assumption of constant intermediate inputs coefficients $\boldsymbol{A}$, labor input coefficients $\boldsymbol{w}_c$, and consumption coefficients $\boldsymbol{h}_c$ yields:

$$\boldsymbol{x} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{h}_c w_{tot} + \boldsymbol{f}^* \qquad (6)$$
$$w_{tot} = \boldsymbol{w}_c'\boldsymbol{x} \qquad (7)$$

- In (7) one could add additional term indicating other exogenous income of HHs (social security benefits, pensions, incomes from financial assets, incomes from the rest of the world). For simplicity, in our discussion of semi-closed IO models this term is ignored.

- The "augmented" total output vector, input coefficients matrix and final demand vector can be, respectively, compactly written as:

$$\overline{\boldsymbol{x}} = \begin{bmatrix} \boldsymbol{x} \\ w_{tot} \end{bmatrix}, \ \overline{\boldsymbol{A}} = \begin{bmatrix} \boldsymbol{A} & \boldsymbol{h}_c \\ \boldsymbol{w}_c' & 0 \end{bmatrix} \text{ and } \overline{\boldsymbol{f}} = \begin{bmatrix} \boldsymbol{f}^* \\ 0 \end{bmatrix}.$$

- Then the expanded IO quantity system in (6)-(7) reads as usual:

$$\overline{\boldsymbol{x}} = \overline{\boldsymbol{A}}\overline{\boldsymbol{x}} + \overline{\boldsymbol{f}} \quad \text{or} \quad \begin{bmatrix} \boldsymbol{x} \\ w_{tot} \end{bmatrix} = \begin{bmatrix} \boldsymbol{A} & \boldsymbol{h}_c \\ \boldsymbol{w}_c' & 0 \end{bmatrix} \begin{bmatrix} \boldsymbol{x} \\ w_{tot} \end{bmatrix} + \begin{bmatrix} \boldsymbol{f}^* \\ 0 \end{bmatrix}$$

- Using the results of partitioned matrices (see e.g. Abadir and Magnus 2005, p. 105), it is easy to derive the expanded Leontief inverse as:

$$(\overline{\boldsymbol{I}} - \overline{\boldsymbol{A}})^{-1} = \begin{bmatrix} \boldsymbol{I} - \boldsymbol{A} & -\boldsymbol{h}_c \\ -\boldsymbol{w}_c' & 1 \end{bmatrix}^{-1} = \begin{bmatrix} \boldsymbol{L} & \boldsymbol{0} \\ \boldsymbol{0}' & 0 \end{bmatrix} + \kappa \begin{bmatrix} \boldsymbol{L}\boldsymbol{h}_c\boldsymbol{w}_c'\boldsymbol{L} & \boldsymbol{L}\boldsymbol{h}_c \\ \boldsymbol{w}_c'\boldsymbol{L} & 1 \end{bmatrix} \quad (8)$$

where $\kappa \equiv \frac{1}{1-\boldsymbol{w}_c'\boldsymbol{L}\boldsymbol{h}_c}$ is basically Keynesian macro-multiplier (see next).

Endogenizing aggregate households

- Thus, (8) and $\overline{\boldsymbol{x}} = (\overline{\boldsymbol{I}} - \overline{\boldsymbol{A}})^{-1}\overline{\boldsymbol{f}}$ together yield this semi-closed IO solution as:

$$\boldsymbol{x} = (\boldsymbol{L} + \kappa \boldsymbol{L}\boldsymbol{h}_c \boldsymbol{w}_c' \boldsymbol{L})\boldsymbol{f}^* = \boldsymbol{L}(\boldsymbol{I} + \kappa \boldsymbol{h}_c \boldsymbol{w}_c' \boldsymbol{L})\boldsymbol{f}^* \quad (9)$$

$$w_{tot} = \kappa \boldsymbol{w}_c' \boldsymbol{L}\boldsymbol{f}^* \quad (10)$$

- Interpretations:
  - $\boldsymbol{w}_c' \boldsymbol{L}\boldsymbol{h}_c$ = direct increase in HHs income resulting from expenditure of extra unit of HHs income; it is the *scalar equivalent* of Miyazawa's "**inter-income-group coefficients**" matrix (to be discussed next).
  - $\kappa \equiv \frac{1}{1 - \boldsymbol{w}_c' \boldsymbol{L}\boldsymbol{h}_c}$ = total increase (direct, indirect and induced) in HHs income resulting from expenditure of an extra unit of HHs income; it is the *scalar equivalent* of Miyazawa's "**interrelational income multiplier**" matrix.
  - Effects of $\boldsymbol{f}^*$ on outputs, (9): the first is the standard Leontief inverse $\boldsymbol{L}$; the second is $(\boldsymbol{I} + \kappa \boldsymbol{h}_c \boldsymbol{w}_c' \boldsymbol{L})$, which increases the initial final demand effect $\boldsymbol{I}\boldsymbol{f}^*$ by $\boldsymbol{h}_c \kappa \boldsymbol{w}_c' \boldsymbol{L}\boldsymbol{f}^*$ due to endogenizing HHs income spending effect: $\boldsymbol{L}\boldsymbol{f}^*$ - initial output without HHs spending $\Rightarrow$ $\boldsymbol{w}_c' \boldsymbol{L}\boldsymbol{f}^*$ - resultant initial HHs income payments $\Rightarrow$ $\kappa \boldsymbol{w}_c' \boldsymbol{L}\boldsymbol{f}^*$ - total HHs income generated, (10), which is translated into consumption demand by $\boldsymbol{h}_c \kappa \boldsymbol{w}_c' \boldsymbol{L}\boldsymbol{f}^*$.

▸ Miyazawa model solution ▸ output multiplier

Looking deeper into the separate effects of the final demand stimulus $\boldsymbol{f}^*$:

"Income rounds"	Output effects (Output increases)	Income effects (Income increases)	Consumption effects (Consump. increases)
1	$\boldsymbol{L}\boldsymbol{f}^*$	$\boldsymbol{w}_c'\boldsymbol{L}\boldsymbol{f}^*$	$\boldsymbol{h}_c\boldsymbol{w}_c'\boldsymbol{L}\boldsymbol{f}^*$
2	$\boldsymbol{L}\boldsymbol{h}_c\boldsymbol{w}_c'\boldsymbol{L}\boldsymbol{f}^*$	$(\boldsymbol{w}_c'\boldsymbol{L}\boldsymbol{h}_c)\boldsymbol{w}_c'\boldsymbol{L}\boldsymbol{f}^*$	$\boldsymbol{h}_c(\boldsymbol{w}_c'\boldsymbol{L}\boldsymbol{h}_c)\boldsymbol{w}_c'\boldsymbol{L}\boldsymbol{f}^*$
3	$\boldsymbol{L}\boldsymbol{h}_c(\boldsymbol{w}_c'\boldsymbol{L}\boldsymbol{h}_c)\boldsymbol{w}_c'\boldsymbol{L}\boldsymbol{f}^*$	$(\boldsymbol{w}_c'\boldsymbol{L}\boldsymbol{h}_c)(\boldsymbol{w}_c'\boldsymbol{L}\boldsymbol{h}_c)\boldsymbol{w}_c'\boldsymbol{L}\boldsymbol{f}^*$ $= (\boldsymbol{w}_c'\boldsymbol{L}\boldsymbol{h}_c)^2\boldsymbol{w}_c'\boldsymbol{L}\boldsymbol{f}^*$	$\boldsymbol{h}_c(\boldsymbol{w}_c'\boldsymbol{L}\boldsymbol{h}_c)^2\boldsymbol{w}_c'\boldsymbol{L}\boldsymbol{f}^*$
4	$\boldsymbol{L}\boldsymbol{h}_c(\boldsymbol{w}_c'\boldsymbol{L}\boldsymbol{h}_c)^2\boldsymbol{w}_c'\boldsymbol{L}\boldsymbol{f}^*$	$(\boldsymbol{w}_c'\boldsymbol{L}\boldsymbol{h}_c)(\boldsymbol{w}_c'\boldsymbol{L}\boldsymbol{h}_c)^2\boldsymbol{w}_c'\boldsymbol{L}\boldsymbol{f}^*$ $=(\boldsymbol{w}_c'\boldsymbol{L}\boldsymbol{h}_c)^3\boldsymbol{w}_c'\boldsymbol{L}\boldsymbol{f}^*$	$\boldsymbol{h}_c(\boldsymbol{w}_c'\boldsymbol{L}\boldsymbol{h}_c)^3\boldsymbol{w}_c'\boldsymbol{L}\boldsymbol{f}^*$
$\vdots$	$\vdots$	$\vdots$	$\vdots$
s	$\boldsymbol{L}\boldsymbol{h}_c(\boldsymbol{w}_c'\boldsymbol{L}\boldsymbol{h}_c)^{s-2}\boldsymbol{w}_c'\boldsymbol{L}\boldsymbol{f}^*$	$(\boldsymbol{w}_c'\boldsymbol{L}\boldsymbol{h}_c)^{s-1}\boldsymbol{w}_c'\boldsymbol{L}\boldsymbol{f}^*$	$\boldsymbol{h}_c(\boldsymbol{w}_c'\boldsymbol{L}\boldsymbol{h}_c)^{s-1}\boldsymbol{w}_c'\boldsymbol{L}\boldsymbol{f}^*$
$\vdots$	$\vdots$	$\vdots$	$\vdots$
Sum	$\boldsymbol{L}\boldsymbol{f}^* + \boldsymbol{L}\boldsymbol{h}_c(\kappa)\boldsymbol{w}_c'\boldsymbol{L}\boldsymbol{f}^*$	$\kappa\boldsymbol{w}_c'\boldsymbol{L}\boldsymbol{f}^*$	$\boldsymbol{h}_c(\kappa)\boldsymbol{w}_c'\boldsymbol{L}\boldsymbol{f}^*$

- The previous IO model with endogenous HHs of one type is a particular case of Miyazawa's (1976) model

Visualising IOT with endogenous disaggregated HHs income groups

	Interm. demand of industries	Disaggregated HHs consumption	Other final uses	Totals
Industries sales	$\boldsymbol{Z}$	$\boldsymbol{H}$	$\boldsymbol{f}^*$	$\boldsymbol{x}$
Disaggregated HHs outputs	$\boldsymbol{W}$	$\boldsymbol{O}$	$\mathbf{0}$	$\boldsymbol{w}_{tot}$
Total imports	$\boldsymbol{m}'$	$\boldsymbol{m}_h'$	m_{f*}	m_{tot}
Other GVA	$(\boldsymbol{v}^*)'$	$\boldsymbol{v}_h'$	v_{f*}	v_{tot}^*
Totals	$\boldsymbol{x}'$	$\boldsymbol{h}_{tot}'$	f_{tot}^*	

- With HHs separated into r distinct income groups
  - $\boldsymbol{H}_c \equiv \boldsymbol{H}\hat{\boldsymbol{w}}_{tot}^{-1}$ is the $n \times r$ matrix of consumption coefficients
  - $\boldsymbol{W}_c \equiv \boldsymbol{W}\hat{\boldsymbol{x}}^{-1}$ is the $r \times n$ matrix of HH inputs coefficients

Semi-closed IO

- Miyazawa's matrix **consumption function** is: $\boldsymbol{H}_f = \boldsymbol{H}_c \boldsymbol{W}_c \hat{\boldsymbol{x}}$
- Plugging this consumption function in the semi-closed IO model

$$\boldsymbol{x} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{H}_f \boldsymbol{\imath} + \boldsymbol{f}^* = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{H}_c \boldsymbol{W}_c \boldsymbol{x} + \boldsymbol{f}^*,$$

which if solved for $\boldsymbol{x}$ yields (Miyazawa 1976, p. 5):

Miyazawa model solution

$$\boldsymbol{x} = \boldsymbol{L}(\boldsymbol{I} + \boldsymbol{H}_c \boldsymbol{K} \boldsymbol{W}_c \boldsymbol{L})\boldsymbol{f}^* \tag{11}$$

$$\boldsymbol{w}_{tot} = \boldsymbol{K}\boldsymbol{W}_c \boldsymbol{L} \boldsymbol{f}^* \tag{12}$$

where $\boldsymbol{W}_c \boldsymbol{L} \boldsymbol{H}_c$ and $\boldsymbol{K} \equiv (\boldsymbol{I} - \boldsymbol{W}_c \boldsymbol{L} \boldsymbol{H}_c)^{-1}$ are, respectively, the matrices of "inter-income-group coefficients" and "interrelational income multiplier".

- Eqs. (11) and (12) are generalizations of (9) and (10), respectively
- Extended IO models: Batey et al. (1987), Hewings et al. (2001), Miller and Blair (2009), Chen et al. (2016), Oosterhaven (2019)

semi-closed IO solution

The j-th column sum of the Leontief inverse gives the total (extra) output in euros in all industries per one (extra) euro of final demand in industry j. It is referred to as the output multiplier of industry j.

- Useful in (short-run) impact analyses: economy-wide impact of changes in final demand components: e.g. government spending
- Type I output multipliers: capture the initial, direct and indirect effects

$$\boldsymbol{m}_I' = \imath' \boldsymbol{L} \tag{13}$$

- Type II output multipliers: capture the initial, direct, indirect and induced effects within the standard semi-closed IO model (9):

$$\boldsymbol{m}_{II}' = \imath' \boldsymbol{L}(\boldsymbol{I} + \kappa \boldsymbol{h}_c \boldsymbol{w}_c' \boldsymbol{L}) = \boldsymbol{m}_I' + \kappa \boldsymbol{m}_I' \boldsymbol{h}_c \boldsymbol{w}_c' \boldsymbol{L} \tag{14}$$

- Exercise: Calculate $\boldsymbol{m}_I$ and $\boldsymbol{m}_{II}$ using the 2015 Spanish IOT

Policy-relevant IO multipliers

- Strictly speaking, gross output is not so important policy variable, though useful for understanding structural change in an economy
- Main policy goals: income generation, job creation, reduction of CO_2 emissions, etc.
- In general, one could focus on diverse **factors** associated with interindustry activities:
  - Economic variables: employment, capital, imports
  - Energy consumption: crude oil, coal, other energy products
  - Use of (other) natural resources: water, land, forest, minerals, metals
  - Environmental burdens: emissions of greenhouse gases, other air pollutants, general waste, toxic compounds in soil and water, affluent discharge into the ocean
  - Considering these factors within IO models is straightforward
- For detailed discussions of different types of generalized IO multipliers (backward and forward linkages), see e.g. Lenzen (2001) and Temurshoev and Oosterhaven (2014).

Policy-relevant IO multipliers

- Let $\boldsymbol{P}$ denote the $p \times n$ matrix of p "factors" used/generated in the production process of each of the n industries
  - E.g., row 1 could indicate industries' *carbon emissions*
  - Row 2 - *number of workers* employed in each industry, etc.
- These factors need be linked to industries' outputs through the direct factor coefficients matrix $\boldsymbol{D}$, whose typical element is defined as:

$$d_{kj} = \frac{p_{kj}}{x_j} = \frac{\text{Factor } k \text{ used/generated in/by industry } j}{\text{Total output of industry } j}$$

- Hence, the industrial consumption/production of factors can be equally derived as $p_{kj} = d_{kj}x_j$, or $\boldsymbol{P} = \boldsymbol{D}\hat{\boldsymbol{x}}$
- Total economy-wide use/generation of factor k is $p_k = \sum_{j=1}^{n} p_{kj}$, or $\boldsymbol{p} = \boldsymbol{D}\boldsymbol{x}$. Using the quantity IO model (5) for $\boldsymbol{x}$ yields:

$$\boldsymbol{p} = \boldsymbol{DLf} \qquad (15)$$

- From (15), the total factor consumption/production multiplier matrix is defined as $\boldsymbol{M_I} = \boldsymbol{DL}$:
  Interpretation: m_{kj} is the amount of factor k required/used/generated in the production process of all industries necessary to satisfy one (extra) euro final demand in industry j.
- If endogenizing HHs is relevant for the factor impact analysis, Type II factor consumption/production multiplier matrix, which *additionally* accounts for the induced effects, is: $\boldsymbol{M_{II}} = \boldsymbol{DL}(\boldsymbol{I} + \kappa \boldsymbol{h_c} \boldsymbol{w_c'} \boldsymbol{L}) = \boldsymbol{M_I} + \kappa \boldsymbol{M_I} \boldsymbol{h_c} \boldsymbol{w_c'} \boldsymbol{L}$

A final note on multipliers:

- Type I multipliers probably underestimate economic impacts: household activity is ignored
- Type II possibly overestimate the impact: stringent assumptions on labor income and the associated consumer spending
- Oosterhaven et al. (1986, p. 69):
  "These two multipliers may be considered as upper and lower bounds on the true indirect effect of an increase in final demand; a realistic estimate generally lies roughly halfway between the Type I and Type II multipliers."

Multi-region IO (MRIO) models

- MRIO models explicitly consider regions, countries
- Could include (endogenize) the rest of the world (RoW) region
- World IO models: the entire world as one country with many regions
- Possible terminology confusion: IRIO (full-survey) vs. MRIO (non-survey, based on fixed trade proportions)

Advantages

- More realistic: incorporates the regional heterogeneity, and interregional dependencies
- Globalization, production fragmentation, global value chains, import-contents of exports, etc.

Disadvantages

- More data intensive, hence could be less transparent in analysis
- More data uncertainty: interregional trade data and other data for the missing regions are mainly estimated (non-survey data)

Figure 2 Schematic outline of World Input-Output Table (WIOT), three regions

		Country A Intermediate *Industry*	Country B Intermediate *Industry*	Rest of World Intermediate *Industry*	Country A Final domestic	Country B Final domestic	Rest of World Final domestic	Total
Country A	*Industry*	Intermediate use of domestic output	Intermediate use by B of exports from A	Intermediate use by RoW of exports from A	Final use of domestic output	Final use by B of exports from A	Final use by RoW of exports from A	Output in A
Country B	*Industry*	Intermediate use by A of exports from B	Intermediate use of domestic output	Intermediate use by RoW of exports from B	Final use by A of exports from B	Final use of domestic output	Final use by RoW of exports from B	Output in B
Rest of World (RoW)	*Industry*	Intermediate use by A of exports from RoW	Intermediate use by B of exports from RoW	Intermediate use of domestic output	Final use by A of exports from RoW	Final use by B of exports from RoW	Final use of domestic output	Output in RoW
		Value added	Value added	Value added				
		Output in A	Output in B	Output in RoW				

Source: Timmer et al. (2012), The world input-output database (WIOD): contents, sources and methods, *WIOD Working Paper* 10 (p.63)

$$\begin{pmatrix} \boldsymbol{x}^1 \\ \boldsymbol{x}^2 \\ \vdots \\ \boldsymbol{x}^p \end{pmatrix} = \begin{pmatrix} \boldsymbol{I}_{n_1} - \boldsymbol{A}^{11} & -\boldsymbol{A}^{12} & \cdots & -\boldsymbol{A}^{1p} \\ -\boldsymbol{A}^{21} & \boldsymbol{I}_{n_2} - \boldsymbol{A}^{22} & \cdots & -\boldsymbol{A}^{2p} \\ \vdots & \vdots & \ddots & \vdots \\ -\boldsymbol{A}^{p1} & -\boldsymbol{A}^{p2} & \cdots & \boldsymbol{I}_{n_p} - \boldsymbol{A}^{pp} \end{pmatrix}^{-1} \begin{pmatrix} \boldsymbol{f}^1 \\ \boldsymbol{f}^2 \\ \vdots \\ \boldsymbol{f}^p \end{pmatrix} \tag{16}$$

where:
$\boldsymbol{A}^{rr}$ the (intra)regional input coefficients matrix for region r $(= 1, \dots, p)$,
$\boldsymbol{A}^{rs}$ the matrix of interregional input (or trade) coefficients with deliveries from region r to region s $(r \neq s)$,
$\boldsymbol{f}^r$ final demand of region r,
$\boldsymbol{x}^r$ gross output of region r, and
$\boldsymbol{I}_{n_r}$ identity matrix of dimension n_r (number of sectors in region r)

Note: Each region may have different number of industries, i.e. in general $n_r \neq n_s$ for $r \neq s$.

Notation matter!

- In the Renaissance, mathematics was written in a verbal style with p for plus, m for minus and R for square root. So, when Gerolamo Cardano (1501–1576) writes

  $$5p : Rm : 15$$
  $$5m : Rm : 15$$
  $$25m : m : 15 \text{ qd est } 40,$$

  what does he mean?

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$$5\text{p} : \text{Rm} : 15$$
$$5\text{m} : \text{Rm} : 15$$
$$25\text{m} : \text{m} : 15 \text{ qd est } 40,$$

  what does he mean?
- He means: $(5 + \sqrt{-15})(5 - \sqrt{-15}) = 25 - (-15) = 40.$
- "There is no doubt that the development of good notation has been of great importance in the history of mathematics" (Abadir and Magnus, 2002, *Econometrics Journal*, volume 5).

Notation and matrix algebra basics

- Matrices: bold capitals; vectors: bold lower cases; scalars: lower cases
- $\mathbb{R}$ is the set of real numbers; $\mathbb{R}^2 = \mathbb{R} \times \mathbb{R}$ denotes the real plane
- $\mathbb{R}^n$ is the set of real $n \times 1$ vectors
- $\boldsymbol{x}$ denotes a column vector. If $\boldsymbol{x} \in \mathbb{R}^n$, then:

$$\boldsymbol{x} = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix}$$

- $\boldsymbol{x}'$ is a row vector (transposition is indicated by a prime)
- $\hat{\boldsymbol{x}}$ is a diagonal matrix, i.e.

$$\hat{\boldsymbol{x}} = \begin{pmatrix} x_1 & 0 & \dots & 0 \\ 0 & x_2 & \dots & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \dots & x_n \end{pmatrix}$$

Notation and matrix algebra basics

- $\mathbb{R}^{m \times n}$ is the set of real $m \times n$ matrices
- Let $\boldsymbol{B} = (b_{ij}) \in \mathbb{R}^{m \times n}$. Then:

$$\boldsymbol{B} = \begin{pmatrix} b_{11} & b_{12} & \dots & b_{1n} \\ b_{21} & b_{22} & \dots & b_{2n} \\ \vdots & \vdots & & \vdots \\ b_{m1} & b_{m2} & \dots & b_{mn} \end{pmatrix}$$

- Consider two matrices: $\boldsymbol{B} \in \mathbb{R}^{m \times k}$ and $\boldsymbol{C} \in \mathbb{R}^{k \times n}$.
- Then $\boldsymbol{BC}$ is an $m \times n$ matrix, with its typical entry in row i and column j defined as

$$(\boldsymbol{BC})_{ij} = \boldsymbol{B}_{i\bullet}\boldsymbol{C}_{\bullet j} = \sum_{h=1}^{k} b_{ih}c_{hj}$$

- Transposition property: $(\boldsymbol{BC})' = \boldsymbol{C}'\boldsymbol{B}'$

- Addition laws:
  - Commutative law: $\boldsymbol{A}+\boldsymbol{B}=\boldsymbol{B}+\boldsymbol{A}$
  - Distributive law: $c(\boldsymbol{A}+\boldsymbol{B})=c\boldsymbol{B}+c\boldsymbol{A}$
  - Associative law: $\boldsymbol{A}+(\boldsymbol{B}+\boldsymbol{C})=(\boldsymbol{A}+\boldsymbol{B})+\boldsymbol{C}$
- Multiplication laws:
  - Commutative law is *usually broken*: $\boldsymbol{AB}\neq\boldsymbol{BA}$
  - Distributive law from the left: $\boldsymbol{C}(\boldsymbol{A}+\boldsymbol{B})=\boldsymbol{CB}+\boldsymbol{CA}$
  - Distributive law from the right: $(\boldsymbol{A}+\boldsymbol{B})\boldsymbol{C}=\boldsymbol{BC}+\boldsymbol{AC}$
  - Associative law for $\boldsymbol{ABC}$ (*parentheses not needed!*): $\boldsymbol{A}(\boldsymbol{BC})=(\boldsymbol{AB})\boldsymbol{C}$

  - Matrix powers follow the same rules as numbers:
    $\boldsymbol{A}^p=\underbrace{\boldsymbol{AAA}\cdots\boldsymbol{A}}_{p\text{ factors}}$
    $(\boldsymbol{A}^p)(\boldsymbol{A}^q)=\boldsymbol{A}^{p+q}$
    $(\boldsymbol{A}^p)^q=\boldsymbol{A}^{pq}$

If there exists an $n \times n$ matrix $\boldsymbol{B}$ such that

$$\boldsymbol{AB} = \boldsymbol{BA} = \boldsymbol{I}_n, \tag{17}$$

then $\boldsymbol{B}$ is called the (two-sided) **inverse** of $\boldsymbol{A}$, denoted by $\boldsymbol{A}^{-1}$.

- If the matrix is invertible, then its inverse is unique (its left-inverse and right-inverse must be the same matrix)
- If $\boldsymbol{A}$ and $\boldsymbol{B}$ are invertible, then so is $\boldsymbol{AB}$.
- Assume there is a nonzero vector $\boldsymbol{x}$ such that $\boldsymbol{Ax} = \mathbf{0}$. Then $\boldsymbol{A}$ cannot have an inverse. No matrix can bring $\mathbf{0}$ back to $\boldsymbol{x}$.
- If $\boldsymbol{A}$ is invertible, then $\boldsymbol{Ax} = \mathbf{0}$ can only have the zero solution $\boldsymbol{x} = \boldsymbol{A}^{-1}\mathbf{0} = \mathbf{0}$.
- A matrix is invertible, if its determinant is nonzero. A 2×2 matrix is invertible if $ad - bc \neq 0$:

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}^{-1} = \frac{1}{ad - bc} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix} \tag{18}$$

Properties of invertible matrices

1. $(\boldsymbol{A}^{-1})^{-1} = \boldsymbol{A}$
2. For $c \neq 0$, $(c\boldsymbol{A})^{-1} = \frac{1}{c}\boldsymbol{A}^{-1}$
3. $(\boldsymbol{ABC})^{-1} = \boldsymbol{C}^{-1}\boldsymbol{B}^{-1}\boldsymbol{A}^{-1}$ (reverse order)
4. $(\boldsymbol{A}')^{-1} = (\boldsymbol{A}^{-1})'$
5. $(\boldsymbol{A}^{n})^{-1} = (\boldsymbol{A}^{-1})^{n}$ for all nonnegative integers n.

Definition

If $\boldsymbol{A}$ is an invertible matrix and n is a positive integer, then $\boldsymbol{A}^{-n}$ is defined by $\boldsymbol{A}^{-n} = (\boldsymbol{A}^{-1})^{n} = (\boldsymbol{A}^{n})^{-1}$

Exercise (optional)

Solve the following matrix equation for $\boldsymbol{X}$ (assuming that all the indicated operations are defined):

$$(\boldsymbol{XB})^{-1}\boldsymbol{A}^{-1} = (\boldsymbol{B}^{2}\boldsymbol{A}^{-1})^{2}$$

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