Mixed and combined input-output models

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Motivation for the mixed IO models

"In certain situations a mixed type of input-output model may be appropriate, in which final demands for some sectors and gross outputs for the remaining sectors are specified exogenously. For example, due to a strike of a major supplier, output from a particular sector might be fixed at the amounts currently on hand in warehouses, awaiting transportation and delivery to buyers. Or, in a planned economy, a target might be to increase agricultural output by 12 percent by the end of the next planning period." (Miller and Blair 2009, p.621)

- Exogenizing outputs of certain industries can be also justified by production quotas, shortages or changes in the available amount of factors of production and resources. For example:

Instead the focus [of the study] is on the economy-wide output and employment effects resulting from changes in the total land area devoted to each of the particular forest types. This aspect is more relevant to policy appraisal, given that forestry policy acts to influence the land area under each forest type, rather than operating to control the output of forestry sectors. (Eiser and Roberts 2002, pp. 70-71)

Mathematics of the standard mixed model

- Without loss of generality, assume that in an n-industry model the first $k \in [2, n-1]$ gross outputs and the last $(n-k)$ final demands are **endogenous**, i.e. determined within the IO system.
- Partitioning the linear IO system $(\boldsymbol{I} - \boldsymbol{A})\boldsymbol{x} = \boldsymbol{f}$ into **endogenous** and **exogenous** components of $\boldsymbol{x}$ and $\boldsymbol{f}$ will help to find the relevant multiplier matrix:

$$\begin{bmatrix} \boldsymbol{I} - \boldsymbol{A}_{11} & -\boldsymbol{A}_{12} \\ -\boldsymbol{A}_{21} & \boldsymbol{I} - \boldsymbol{A}_{22} \end{bmatrix} \begin{bmatrix} \boldsymbol{x}_1 \\ \boldsymbol{x}_2 \end{bmatrix} = \begin{bmatrix} \boldsymbol{f}_1 \\ \boldsymbol{f}_2 \end{bmatrix}.$$

Rearrange the above system such that all endogenous variables appear on the left-hand side, while the exogenous variables on the right-hand side:

$$(\boldsymbol{I} - \boldsymbol{A}_{11})\boldsymbol{x}_1 = \boldsymbol{f}_1 + \boldsymbol{A}_{12}\boldsymbol{x}_2 \tag{1a}$$

$$-\boldsymbol{A}_{21}\boldsymbol{x}_1 - \boldsymbol{f}_2 = -(\boldsymbol{I} - \boldsymbol{A}_{22})\boldsymbol{x}_2 \tag{1b}$$

- The system (1) can be more compactly written as follows:

$$\begin{bmatrix} (\boldsymbol{I} - \boldsymbol{A}_{11}) & \boldsymbol{O} \\ -\boldsymbol{A}_{21} & -\boldsymbol{I} \end{bmatrix} \begin{bmatrix} \boldsymbol{x}_1 \\ \boldsymbol{f}_2 \end{bmatrix} = \begin{bmatrix} \boldsymbol{I} & \boldsymbol{A}_{12} \\ \boldsymbol{O} & -(\boldsymbol{I} - \boldsymbol{A}_{22}) \end{bmatrix} \begin{bmatrix} \boldsymbol{f}_1 \\ \boldsymbol{x}_2 \end{bmatrix},$$

  where $\boldsymbol{O}$ is the null matrix of appropriate dimension.

- Solving the above system with respect to the endogenous variables yields:

$$\begin{bmatrix} \boldsymbol{x}_1 \\ \boldsymbol{f}_2 \end{bmatrix} = \begin{bmatrix} (\boldsymbol{I} - \boldsymbol{A}_{11}) & \boldsymbol{O} \\ -\boldsymbol{A}_{21} & -\boldsymbol{I} \end{bmatrix}^{-1} \begin{bmatrix} \boldsymbol{I} & \boldsymbol{A}_{12} \\ \boldsymbol{O} & -(\boldsymbol{I} - \boldsymbol{A}_{22}) \end{bmatrix} \begin{bmatrix} \boldsymbol{f}_1 \\ \boldsymbol{x}_2 \end{bmatrix},$$

  which using the results on the inverse of partitioned matrices (Abadir and Magnus 2005, p. 104) gives:

$$\begin{bmatrix} \boldsymbol{x}_1 \\ \boldsymbol{f}_2 \end{bmatrix} = \begin{bmatrix} \boldsymbol{L}_{11} & \boldsymbol{O} \\ -\boldsymbol{A}_{21}\boldsymbol{L}_{11} & -\boldsymbol{I} \end{bmatrix} \begin{bmatrix} \boldsymbol{I} & \boldsymbol{A}_{12} \\ \boldsymbol{O} & -(\boldsymbol{I} - \boldsymbol{A}_{22}) \end{bmatrix} \begin{bmatrix} \boldsymbol{f}_1 \\ \boldsymbol{x}_2 \end{bmatrix}, \qquad (2)$$

  where $\boldsymbol{L}_{11} \equiv (\boldsymbol{I} - \boldsymbol{A}_{11})^{-1}$ is the $k \times k$ Leontief inverse that includes only the first group of industries.

- Thus, (2) gives the general solution of the mixed IO model

General solution of the mixed IO model

The general solution of the mixed IO model with the exogenous final demand for the first k industries and exogenous gross outputs for the last $n-k$ industries is:

$$\begin{bmatrix} \boldsymbol{x}_1 \\ \boldsymbol{f}_2 \end{bmatrix} = \begin{bmatrix} \boldsymbol{L}_{11} & \boldsymbol{L}_{11}\boldsymbol{A}_{12} \\ -\boldsymbol{A}_{21}\boldsymbol{L}_{11} & (\boldsymbol{I}-\boldsymbol{A}_{22}) - \boldsymbol{A}_{21}\boldsymbol{L}_{11}\boldsymbol{A}_{12} \end{bmatrix} \begin{bmatrix} \boldsymbol{f}_1 \\ \boldsymbol{x}_2 \end{bmatrix}, \quad (3)$$

or, equivalently, in terms of *changes* (Δ):

$$\begin{bmatrix} \Delta\boldsymbol{x}_1 \\ \Delta\boldsymbol{f}_2 \end{bmatrix} = \begin{bmatrix} \boldsymbol{L}_{11} & \boldsymbol{L}_{11}\boldsymbol{A}_{12} \\ -\boldsymbol{A}_{21}\boldsymbol{L}_{11} & (\boldsymbol{I}-\boldsymbol{A}_{22}) - \boldsymbol{A}_{21}\boldsymbol{L}_{11}\boldsymbol{A}_{12} \end{bmatrix} \begin{bmatrix} \Delta\boldsymbol{f}_1 \\ \Delta\boldsymbol{x}_2 \end{bmatrix}. \quad (4)$$

- Ignoring the extreme cases of no exogenous outputs ($k = n$) or all exogenous outputs ($k = 0$), other two extreme cases in (4) are:
  1. $\Delta\boldsymbol{f}_1 \neq \boldsymbol{0}$ with $\Delta\boldsymbol{x}_2 = \boldsymbol{0}$, and
  2. $\Delta\boldsymbol{f}_1 = \boldsymbol{0}$ with $\Delta\boldsymbol{x}_2 \neq \boldsymbol{0}$.

TABLE: Standard and mixed model multipliers, Spain, 2015

	ind1	ind2	ind3	ind4	ind5	ind6	ind7	ind8	ind9	ind10
Standard Leontief inverse (i.e. $k = 10$)										
ind1	1.060	0.070	0.019	0.012	0.007	0.002	0.001	0.007	0.006	0.006
ind2	0.346	1.521	0.308	0.180	0.142	0.044	0.022	0.116	0.105	0.099
ind3	0.012	0.017	1.234	0.021	0.021	0.017	0.040	0.014	0.014	0.014
ind4	0.193	0.173	0.140	1.209	0.088	0.045	0.016	0.122	0.064	0.069
ind5	0.011	0.018	0.015	0.018	1.210	0.032	0.006	0.031	0.023	0.022
ind6	0.029	0.024	0.031	0.031	0.023	1.235	0.056	0.026	0.021	0.021
ind7	0.016	0.019	0.050	0.069	0.062	0.048	1.013	0.041	0.017	0.046
ind8	0.076	0.100	0.126	0.116	0.138	0.110	0.048	1.203	0.061	0.100
ind9	0.007	0.010	0.010	0.014	0.012	0.004	0.002	0.011	1.032	0.007
ind10	0.005	0.008	0.005	0.010	0.008	0.008	0.002	0.019	0.007	1.090
Sum	1.755	1.961	1.938	1.679	1.712	1.545	1.206	1.589	1.350	1.474
Multiplier matrix of the mixed IO model with $k = 7$										
ind1	1.060	0.069	0.018	0.011	0.006	0.002	0.001	0.006	0.005	0.005
ind2	0.338	1.510	0.295	0.167	0.127	0.033	0.017	0.094	0.096	0.082
ind3	0.011	0.016	1.233	0.019	0.020	0.015	0.039	0.011	0.013	0.012
ind4	0.184	0.162	0.126	1.196	0.073	0.034	0.011	0.100	0.056	0.054
ind5	0.009	0.015	0.012	0.015	1.206	0.029	0.005	0.025	0.021	0.018
ind6	0.027	0.022	0.028	0.028	0.020	1.233	0.055	0.021	0.019	0.017
ind7	0.013	0.016	0.045	0.065	0.057	0.044	1.011	0.033	0.014	0.039
ind8	-0.063	-0.082	-0.104	-0.095	-0.114	-0.091	-0.040	0.833	-0.049	-0.076
ind9	-0.006	-0.009	-0.008	-0.012	-0.011	-0.003	-0.001	-0.009	0.969	-0.006
ind10	-0.003	-0.006	-0.003	-0.007	-0.005	-0.006	-0.002	-0.014	-0.005	0.919
Sum ind1-7	1.642	1.810	1.758	1.501	1.509	1.389	1.139			

Interpretation of the typical entry of the multiplier matrix in the mixed model. Assume $i, j \in 1$ and $h, g \in 2$, then:

Δx_i	$(\boldsymbol{L}_{11})_{ij}\Delta f_j$ = industry i's extra output necessary to satisfy Δf_j within the standard k-industry IO model	$(\boldsymbol{L}_{11}\boldsymbol{A}_{12})_{ig}\Delta x_g$ = i's extra output required to allow production of the fixed output change in g, Δx_g
Δf_h	$-(\boldsymbol{A}_{21}\boldsymbol{L}_{11})_{hj}\Delta f_j$ = change in endogenous final demand h needed to satisfy Δf_j, given the fixed outputs of all $g \in 2$ (including h), i.e. with $\Delta x_g = 0$	$(\boldsymbol{I} - \boldsymbol{A}_{22} - \boldsymbol{A}_{21}\boldsymbol{L}_{11}\boldsymbol{A}_{12})_{hg}\Delta x_g$ = change in endogenous final demand h needed to allow production of the fixed output change Δx_g, given the other fixed changes of outputs in group 2

$\boldsymbol{A}_{21}\boldsymbol{L}_{11}\boldsymbol{A}_{12}\Delta \boldsymbol{x}_2$ quantify **inter-group feedback effects** akin to **interregional feedback effects** in an interregional IO (Miller and Blair 2009, pp. 80-81):

- $\boldsymbol{A}_{12}\Delta \boldsymbol{x}_2$ = *direct* inputs from group 1 industries to satisfy $\Delta \boldsymbol{x}_2$,
- $\boldsymbol{L}_{11}\boldsymbol{A}_{12}\Delta \boldsymbol{x}_2$ = *total* (direct plus indirect) inputs from group 1 industries to satisfy $\Delta \boldsymbol{x}_2$, thus
- $\boldsymbol{A}_{21}\boldsymbol{L}_{11}\boldsymbol{A}_{12}\Delta \boldsymbol{x}_2$ = necessary inputs from group 2 industries to satisfy total intermediate demands of group 1 industries due to $\Delta \boldsymbol{x}_2$. Since $\Delta \boldsymbol{x}_2$ is fixed, the feedback effects have to be netted out of group 2 final demands!

New industry/firm impacts

- Assume new industry moves into the region (economy) and it is assumed that *its (planned) production level is known*, next to the regional input requirements to/from this new industry

New industry impacts can be obtained from the mixed IO framework (3) by setting the new industry as a "group 2" industry, i.e.

$$\begin{aligned} \boldsymbol{x}_1 &= \boldsymbol{L}_{11}\boldsymbol{f}_1 + \boldsymbol{L}_{11}\boldsymbol{a}_{1,new}x_{new}, \\ f_{new} &= -\boldsymbol{a}'_{new,1}\boldsymbol{L}_{11}\boldsymbol{f}_1 + (1 - a_{new,new} - \boldsymbol{a}'_{new,1}\boldsymbol{L}_{11}\boldsymbol{a}_{1,new})x_{new}. \end{aligned}$$

For fixed $\boldsymbol{f}_1$, i.e. $\Delta\boldsymbol{f}_1 = \boldsymbol{0}$, extra regional outputs are obtained as:

$$\Delta\boldsymbol{x}_1 = \boldsymbol{L}_{11}\boldsymbol{a}_{1,new}x_{new},$$

which can then be translated to employment and income effects.

- Similar assessment may also be used for assessing the impact of industry close-down or entering/exiting a new/existing firm into/from an existing industry

Alternative approach: output-to-output multipliers

- There is a simpler way of assessing the impact of $\Delta \boldsymbol{x}_2$ on $\Delta \boldsymbol{x}_1$ (or of $\boldsymbol{x}_2$ on $\boldsymbol{x}_1$)

The impacts of exogenous outputs $\boldsymbol{x}_2$ on endogenous outputs $\boldsymbol{x}_1$ can be alternatively found as:

$$\boldsymbol{x}_1 = \boldsymbol{L}^{12}(\boldsymbol{L}^{22})^{-1}\boldsymbol{x}_2, \tag{5}$$

where $\boldsymbol{L}^{12}$ and $\boldsymbol{L}^{22}$ are the corresponding blocks of the standard Leontief inverse of the entire system (i.e. when $k = n$). In particular, if there is only one industry with exogenous (given) output, i.e. $2 = \{g\}$, then

$$x_i = \frac{l_{ig}}{l_{gg}} x_g \text{ for all } i \in 1. \tag{6}$$

- The result (5) is implied by the identity $\boldsymbol{L}^{12}(\boldsymbol{L}^{22})^{-1} = \boldsymbol{L}_{11}\boldsymbol{A}_{12}$ (for a simple proof, see the starting part of the proof of Lemma 2 in Temurshoev 2010, p. 890, first paragraph)

Exercises

1. Consider the aggregated Spanish IOT and assume gross output of industry 10 has increased by 1 mln. EUR. Using the standard 10×10 Leontief inverse presented above, find the impacts of such exogenous output increase on outputs of the remaining nine sectors.
2. Using the alternative output-to-output multiplier approach, replicate the 7×3 submatrix presented in the upper-right part of the mixed model multiplier matrix that shows the effect of a unitary output change in industries 8-10 on outputs of industries 1-7.

Combining IO quantity and price models

- Another criticism that is often raised with regard to IO modeling relates to the fact that within the standard IO modelling approaches interactions between prices and quantities are not allowed
- It should be noted that such link is *not* essential for many applications of IO models: quantifying carbon content of consumption, factor content of trade, etc.
- Econometric IO models deal with all related criticisms adequately, while could be considered superior to traditional CGE models.
- Examples of econometric IO models include:
    - E3ME model of Cambridge Econometrics (endogenous money)
    - INFORUM models (Almon 2017)
    - FIDELIO (Kratena et al. 2013), etc.

Combining IO quantity and price models

- The LINE model (Madsen and Jensen-Butler 2004, Madsen 2008) is one major exception that uses the combined extended IO quantity and price model:
  - Keep sthe richness of very detailed interregional SAM, and
  - Based on the "two-by-two-by-two principle": two sets of actors (producers and institutional units), two markets (commodities and factors) and two locations (origins and destinations)
  - Incorporates commuting and shopping models
- For example, one version of LINE includes
  - Sectors: 12 sectors aggregated from the 133 sectors in the NAs;
  - Factors: 7 age, 2 sex and 5 education groups;
  - Households: 4 types, based upon household composition;
  - Needs: 13 components of private and governmental individual consumption, 8 groups of governmental consumption, and 10 components of gross fixed capital formation;
  - Commodities: 20 commodities (aggregated from 131 in the NAs); and
  - Regions: 277 municipalities, defined either as place of production, place of residence or as place of demand

"*It is the passive role of prices in input-output models which limits the flexibility with which multi-product industries and multi-industry products can be handled... modern applied general equilibrium models are able to incorporate more satisfactory treatments by allowing the commodity compositions of industries' outputs and of final demands to be functions of the relative prices of commodities.*" (Dixon et al. 1992, pp. 44-45)

"*Although CGE models include substitution effects arising from changes in relative prices, determining changes in patterns of demand, CGE models do not generally include a full description of the cost and price determination both in spatial and SAM terms. In general, CGE models do not provide a full description of the operation of the local economy. Furthermore, modelling the complexity of local interregional economy in a CGE framework rapidly leads to problems of derivation of analytical solutions and even when using numerical solution there are problems of mathematical intractability, multiple equilibria and failure to converge on a solution. In addition to these issues, the benefits of the CGE approach are often unclear except perhaps for their anchorage to micro-economic theoretical foundations.*" (Madsen 2008, pp. 186-187)

"Simply put, an economic model is a set of equations which describe how the economy or some part of it functions. In my view, a model should incorporate and test our understanding of how the economy works. Its equations should make sense. And it should be possible to test how adequate our understanding is by running it over the past and seeing how well it can reproduce history. By changing some of its assumptions and rerunning history with the changed assumptions, it is possible to analyze the effects of policies. Finally, it should be useful not only for policy analysis but also for forecasting. By studying the errors of the forecast, the builder of the model may hope to improve his or her understanding of the economy.
...The vector-autoregression (VAR) school gives little or no weight to the equations expressing any sort of understanding of the economy; ability to reproduce the past, however, is of great importance. The computable general equilibrium (CGE) school gives great weight to the equations making sense but has little interest in testing the dynamic properties (if any) of its models or in the equations fitting more than one point. In my view, each of these schools is right in what it values but remiss in what it neglects." (Almon 2017, pp. 11-12)

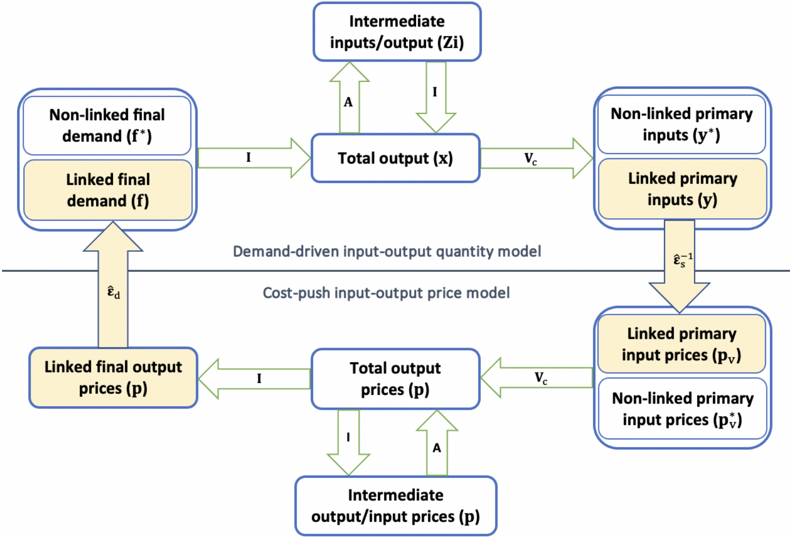
Combined Type I price and quantity model

- The detailed interregional IO quantity and price modules of the LINE model are linked through **iterative switching** between the two submodels (see e.g. Madsen and Jensen-Butler 2004)

> "*The submodels are linked together such that prices influence quantities and quantities influence prices... The links are divided into internal links and external links. Internal links are defined as links, which form a simultaneous system covering both models, whereas external links are links where the links are non-simultaneous, where an endogenous variable, which is an exogenous explaining variable in the other submodel, does not enter as explanatory variable in the submodel itself.*" (Madsen 2008, p. 219)

- Oosterhaven (2019, chap. 5.2) describes the basic details of a combined Type II price and quantity model using the external links approach
- Here we also discuss this perhaps simpler approach to combination of the two IO models

Combined Type I price and quantity model



Source: Own elaborations based on and inspired by Oosterhaven (2019, p. 61).

Combined Type I price and quantity model

- Assume government spending increases by $\Delta \boldsymbol{f}_g$, while industry/product totals of consumption expenditures and exports, $\boldsymbol{f}$, react to price changes
- Formally, the following computations need to be implemented until convergence of the **iterative solution**: changes approaching zero, or totals approaching fixed values

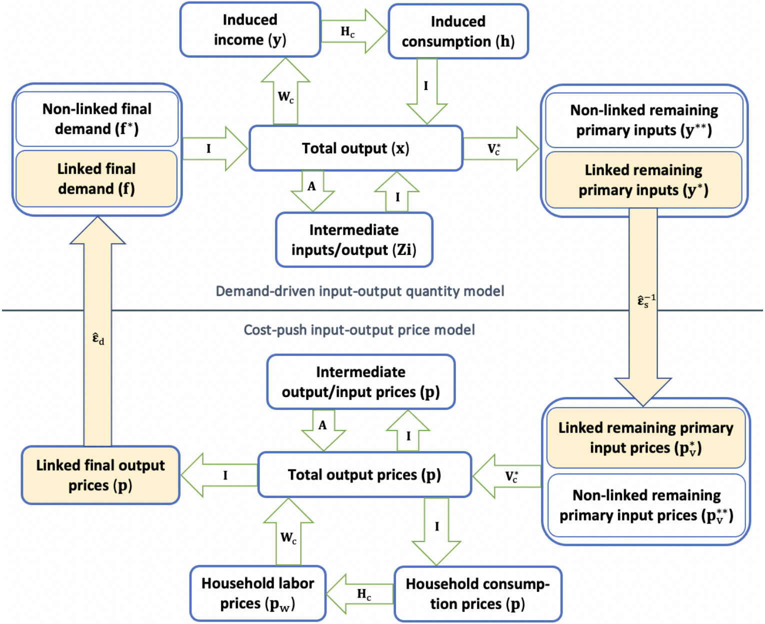
- Compute the variables and their changes of the first step/iteration:
  - Gross outputs: $\Delta \boldsymbol{x}_1 = \boldsymbol{L}\Delta \boldsymbol{f}_g$ and $\boldsymbol{x}_1 = \boldsymbol{x}_0 + \Delta \boldsymbol{x}_1$
  - Total primary inputs: $\Delta \boldsymbol{y}_1 = \boldsymbol{V}_c \Delta \boldsymbol{x}_1$ and $\boldsymbol{y}_1 = \boldsymbol{y}_0 + \Delta \boldsymbol{y}_1$
  - Relative prices of primary inputs: $\Delta \boldsymbol{p}_{v,1} = \hat{\boldsymbol{\varepsilon}}_s^{-1} \hat{\boldsymbol{y}}_0^{-1} \Delta \boldsymbol{y}_1$
  - Relative prices of outputs: $\Delta \boldsymbol{p}_1 = \boldsymbol{L}' \boldsymbol{V}_c' \Delta \boldsymbol{p}_{v,1}$
- Iteratively compute the variables of the subsequent step $k = 2, 3, \ldots, K$:
  - Final demand reaction: $\Delta \boldsymbol{f}_k = \hat{\boldsymbol{f}}_{k-1} \hat{\boldsymbol{\varepsilon}}_d \Delta \boldsymbol{p}_{k-1}$ and $\boldsymbol{f}_k = \boldsymbol{f}_{k-1} + \Delta \boldsymbol{f}_k$
  - Gross outputs: $\Delta \boldsymbol{x}_k = \boldsymbol{L}\Delta \boldsymbol{f}_k$ and $\boldsymbol{x}_k = \boldsymbol{x}_{k-1} + \Delta \boldsymbol{x}_k$
  - Total primary inputs: $\Delta \boldsymbol{y}_k = \boldsymbol{V}_c \Delta \boldsymbol{x}_k$ and $\boldsymbol{y}_k = \boldsymbol{y}_{k-1} + \Delta \boldsymbol{y}_k$
  - Relative prices of primary inputs: $\Delta \boldsymbol{p}_{v,k} = \hat{\boldsymbol{\varepsilon}}_s^{-1} \hat{\boldsymbol{y}}_{k-1}^{-1} \Delta \boldsymbol{y}_k$
  - Relative prices of outputs: $\Delta \boldsymbol{p}_k = \boldsymbol{L}' \boldsymbol{V}_c' \Delta \boldsymbol{p}_{v,k}$

Combined Type I price and quantity model

- The final results imply two new IOTs: one expressed in base year prices and another in new prices
  - Allows separating the impacts of quantities and prices
  - Exercise: Go through the solved example based on the 2015 aggregated IOT of Spain
- The price elasticities of supply of primary inputs ε_s and the price elasticities of final demand ε_d play critically important role in the combined IO model
- In real applications, it is recommended to account for the **uncertainty** around the estimates of these elasticities
- Running Monte Carlo simulations is straightforward, which imply final results with uncertainty indicators (e.g. standard deviation, 'confidence intervals', boxplots, etc.)

Combined Type II price and quantity model

- Type II price and quantity models can be combined in a similar way
- Recall that compared with Type I models, Type II price and quantity models additionally account for, respectively:
  - the income-induced consumption effects, and
  - the cost-push price-wage-price inflationary processes.
- Since consumption expenditures and compensation of employees are already endogenous in Type II models, one way to combine the quantity and price models is by linking:
  - the remaining appropriate final demand components (e.g. exports) to output price changes, and
  - the prices of the appropriate remaining primary inputs (e.g. imports and operating surplus) to their supply.
- Similar to the Type I combined model, using the corresponding price elasticities, a procedure of iterative switching between the outcomes of two Type II submodels results in a combined Type II price and quantity model



Source: Own elaborations based on Oosterhaven (2019, p. 61).

Combined Type II price and quantity model

- Again assume government spending increases by $\Delta \boldsymbol{f}_g$, while only total exports, $\boldsymbol{f}$, react to price changes

- Compute the variables and their changes of the first step/iteration:
  - Gross outputs: $\Delta \boldsymbol{x}_1 = \boldsymbol{L}\Delta \boldsymbol{f}_g$ and $\boldsymbol{x}_1 = \boldsymbol{x}_0 + \Delta \boldsymbol{x}_1$
  - Induced income and consumption: $\boldsymbol{y}_1 = \boldsymbol{W}_c \boldsymbol{x}_1$ and $\boldsymbol{h}_1 = \boldsymbol{H}_c \boldsymbol{y}_1$
  - Remaining primary inputs: $\Delta \boldsymbol{y}_1^* = \boldsymbol{V}_c^* \Delta \boldsymbol{x}_1$ and $\boldsymbol{y}_1^* = \boldsymbol{y}_0^* + \Delta \boldsymbol{y}_1^*$
  - Relative prices of primary inputs: $\Delta \boldsymbol{p}_{v,1}^* = \hat{\varepsilon}_s^{-1} (\hat{\boldsymbol{y}}_0^*)^{-1} \Delta \boldsymbol{y}_1^*$
  - Relative prices of outputs: $\Delta \boldsymbol{p}_1 = \boldsymbol{L}'(\boldsymbol{V}_c^*)' \Delta \boldsymbol{p}_{v,1}^*$
  - Relative prices of labor: $\Delta \boldsymbol{p}_{w,1} = \Delta \boldsymbol{p}_1' \boldsymbol{H}_c$

- Iteratively compute the variables of the subsequent step $k = 2, 3, \ldots, K$:
  - Final demand reaction: $\Delta \boldsymbol{f}_k = \hat{\boldsymbol{f}}_{k-1} \hat{\varepsilon}_d \Delta \boldsymbol{p}_{k-1}$ and $\boldsymbol{f}_k = \boldsymbol{f}_{k-1} + \Delta \boldsymbol{f}_k$
  - Gross outputs: $\Delta \boldsymbol{x}_k = \boldsymbol{L}\Delta \boldsymbol{f}_k$ and $\boldsymbol{x}_k = \boldsymbol{x}_{k-1} + \Delta \boldsymbol{x}_k$
  - Induced income and consumption: $\boldsymbol{y}_k = \boldsymbol{W}_c \boldsymbol{x}_{k-1}$ and $\boldsymbol{h}_k = \boldsymbol{H}_c \boldsymbol{y}_k$
  - Remaining primary inputs: $\Delta \boldsymbol{y}_k^* = \boldsymbol{V}_c^* \Delta \boldsymbol{x}_k$ and $\boldsymbol{y}_k^* = \boldsymbol{y}_{k-1}^* + \Delta \boldsymbol{y}_k^*$
  - Relative prices of primary inputs: $\Delta \boldsymbol{p}_{v,k}^* = \hat{\varepsilon}_s^{-1} (\hat{\boldsymbol{y}}_{k-1}^*)^{-1} \Delta \boldsymbol{y}_k^*$
  - Relative prices of outputs: $\Delta \boldsymbol{p}_k = \boldsymbol{L}'(\boldsymbol{V}_c^*)' \Delta \boldsymbol{p}_{v,k}^*$
  - Relative prices of labor: $\Delta \boldsymbol{p}_{w,k} = \Delta \boldsymbol{p}_k' \boldsymbol{H}_c$

- Except for the LINK model, other studies, to our best knowledge, run just one iteration (see e.g. Choi et al. 2010, 2016). However,

> "*Their interpretation that such an analysis produces the short-run impacts..., while doing more iterations would give an indication of the longer run impacts if false, as the length of the market equilibrium process has no relation with the length of iterative solution of the combined model. In fact, [it] still represents a comparative static model. The duration of the market equilibrium process may be quick if people and firms have perfect expectations about future price and quantity changes. It may also take a long time with temporary adaptations in various levels of stocks when information about future changes in not perfect (Romanoff and Levine 1986).*" Oosterhaven (2019, p. 65)

- Supply-side *quantity* shocks can be simulated in a **joint mixed and combined IO setting** that allows for *both* imposition of exogenous output supply and the interplay between prices and quantities.
- Surís-Regueiro and Santiago (2018) is a one iteration version of such a study
- However, on the grounds of the justifications given earlier, it is recommended to run as many iterations as necessary to reach convergence
- Note: in general in combined IO models, convergence is *not* always guaranteed; however, for reasonable/realistic values of price elasticities convergence should not be an issue!

EXERCISES

1. Write down the formulas of the iterative procedure of the combined Type I price and quantity model when simulating *supply-side price shocks* (such as international oil price hikes, introduction of pollution taxes, imposition of import tariffs, etc).
2. Simulate the effect of a 50% increase in taxes less subsidies on products used by industries using the combined Type I price and quantity model. Compare your results with those derived from the standard Type I price model (see the Excel file of the course exercises).
3. Write down the formulas of the iterative procedure of the combined Type II price and quantity model when simulating *supply-side price shocks*.
4. Simulate the effect of a 50% increase in taxes less subsidies on products used by industries using the combined Type II price and quantity model. Compare your results with those derived from the standard Type II price model (see the Excel file of the course exercises).
5. (Hard!) Try to find the expressions of the iterative procedure in a joint mixed and combined IO setting to simulate the *supply-side quantity shocks*, such as implied by the resources supply constraints or shocks caused by natural or man-made disasters.

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