Global income multipliers: A Miyazawa analysis

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MIYAZAWA's contributions were essential in better understanding the issues of income distribution within the extended input-output (IO) and related economic modeling frameworks. As noted in Hewings et al. (1999, pp. 1-2):

> "Miyazawa's work played an important, although until very recently, underappreciated role in demonstrating one of the first stages along the path towards more sophistication in modeling… In many ways, his work in linking income distribution impacts to input-output systems may be seen as a parallel development to Stone's work in the creation of social accounting systems."

Indeed, all economists are well aware of the *Keynesian macro-multiplier*, yet many are much less aware of its essential generalization to a multi-industry setting due to Japanese economist Ken'ichi Miyazawa (1925-2010). In this essay we apply Miyazawa's concepts of "*interrelational income multiplier*" and "*matrix multiplier of income formation*" to the world input-output tables (WIOTs) available from the World Input-Output Database (WIOD).

For presentation purposes, WIOD's 43 countries plus the rest of the world (RoW) region are aggregated into the following seven groups of countries: ABIIRT - Australia, Brazil, Indonesia, India, Russia and Turkey (which could be thought of as a "resource-rich countries" group); CHN - China; JKT - Japan, Republic of Korea and Taiwan; EU - the current 27 EU member countries; NSUK - Norway, Switzerland and the United Kingdom; USMCA - the United States, Mexico and Canada; and the RoW region itself.[1] The relative size of the economies of these regions in terms of their GDP for the year of 2014 is shown in Figure 1, relative to the world GDP that amounted to $76,988 bln in that year.[2] The figure shows that in 2014 all the 43 WIOD countries, excluding the RoW region, overall accounted for 86% of the world GDP.

For a general overview, the 2014 aggregate WIOT of these seven aggregated regions is presented in Table 1. For easier readability purposes, the interindustry intermediate transactions are shown in orange, final demand flows in yellow, international trade and transport margins

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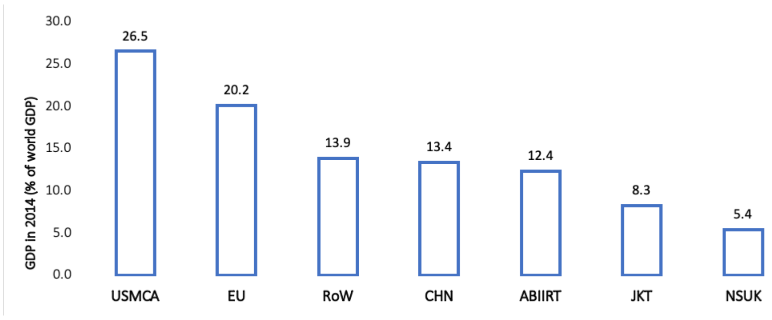
[1] However, in order to avoid any loss of information, all the calculations are done using the full WIOD dataset, which includes e.g. 56 industries for each of the 43 covered countries and the RoW region (see Timmer et al., 2015).

[2] The last WIOD Release 2016 covers the time period from 2000 to 2014.

Figure 1: GDP by regions (% of world GDP), 2014



Source: Own elaborations based on WIOD Release 2016.

(ITM) in green, taxes less subsidies on products (TLS) and gross value added (GVA) in pink, while the WIOT row and column totals in light gray. An "ideal" WIOT would have instead incorporated the ITM row into the orange and yellow blocks by re-distributing the ITM entries to the (world) trade and transport industries supplying these services.[3] Notice a negative number for TLS in the USMCA region, i.e. $-19 bln, which mainly consists of subsidies provided by the Mexican government to most of its industries, with particularly large subsidies given to the land and air transport industries, generation of electricity, gas and steam, construction, and administrative and support service activities.[4]

Table 1: Aggregate world input-output table, 2014 (bln. US$)

	Intermediate demand							Final demand							Output at basic prices
	ABIIRT	CHN	JKT	EU	NSUK	USMCA	RoW	ABIIRT	CHN	JKT	EU	NSUK	USMCA	RoW	
ABIIRT	7,231	168	156	236	38	94	615	8,403	23	24	83	24	54	250	17,401
CHN	119	19,972	174	163	27	177	551	137	9,348	144	151	36	258	487	31,745
JKT	102	294	5,779	83	17	171	382	55	124	5,833	50	14	122	268	13,293
EU	217	125	86	12,785	381	273	784	174	124	72	12,989	278	215	574	29,077
NSUK	38	21	26	360	2,906	96	254	31	22	14	152	3,390	59	149	7,517
USMCA	83	84	105	316	68	14,748	468	45	53	48	109	46	19,863	317	36,354
RoW	502	681	579	596	98	453	11,867	175	123	113	226	62	225	9,910	25,610
ITM	86	115	108	203	34	109	0	40	34	32	106	28	82	0	975
TLS	397	0	45	412	152	-19	0	494	0	94	1,181	229	196	0	3,181
GVA	8,627	10,284	6,236	13,924	3,796	20,251	10,689								73,807
Output	17,401	31,745	13,293	29,077	7,517	36,354	25,610	9,553	9,850	6,375	15,049	4,107	21,074	11,955	

Note: Region abbreviations are defined in the text. TLS = taxes less subsidies on products, GVA = gross value added at basic prices, and ITM = international transport margins. Source: Own elaborations based on WIOD Release 2016.

In order to calculate Miyazawa's interregional income multiplier matrices, the semi-closed interregional IO quantity model in the spirit of Miyazawa (1968) is used, where in addition to intermediate requirements (demands) of industries, final consumption expenditure by households and compensation of employees are endogenized.[5] For the mathematics and detailed overview of the corresponding Miyazawa multipliers in an interregional setting, the reader is

[3] For details on this issue, see Streicher and Stehrer (2015).

[4] According to the WIOD database, the overall TLS in Canada, Mexico and the US in 2014 amounted to $1,494 mln, $-20,484 mln and $-39 mln US, respectively. In the US the largest subsidized industries are air transportation and agriculture.

[5] The data on compensation of employees comes from the WIOD Socio-Economic Accounts. Because compensation of employees is not available for the RoW region, we have estimated this missing data using the aggregate industry GVA and compensation of employees for Brazil, China, India, Indonesia, and Mexico. This is in line with the WIOD approach to modeling the RoW region in WIOTs (Dietzenbacher et al., 2013).

Table 2: Miyazawa's interrelational-interregional income multipliers, 2014

Region of income receipt	Region of income origin							Total (all)	Total (other)
	ABIIRT	CHN	JKT	EU	NSUK	USMCA	RoW		
ABIIRT	1.768	0.011	0.027	0.020	0.019	0.017	0.062	1.924	0.156
CHN	0.062	1.467	0.050	0.030	0.030	0.044	0.120	1.802	0.335
JKT	0.030	0.013	1.617	0.012	0.014	0.026	0.070	1.782	0.164
EU	0.080	0.019	0.036	1.593	0.151	0.053	0.159	2.093	0.499
NSUK	0.022	0.005	0.012	0.035	1.575	0.021	0.058	1.729	0.154
USMCA	0.053	0.018	0.045	0.048	0.056	2.266	0.165	2.650	0.385
RoW	0.105	0.031	0.080	0.054	0.056	0.062	1.615	2.003	0.389
Total (all regions)	2.120	1.565	1.867	1.792	1.902	2.488	2.250	13.984	
Total (other regions)	0.352	0.098	0.250	0.199	0.327	0.222	0.635		2.082

referred to the Appendix of this essay. Table 2 shows Miyazawa's interrelational income multipliers, formalized as matrix **K** in equation (9b) in the Appendix.[6]

The rs-th entry of the interrelational income multiplier matrix indicates the total household income in region r generated (or induced) by expenditures from $1 of income initially earned in region s.[7] Take, for example, the fifth column corresponding to the USMCA region in Table 2: $1 of income earned in USMCA generates a total of $2.266 in USMCA itself (this includes the original $1 initially induced income), $0.062 in RoW, $0.053 in EU, $0.044 in CHN, $0.026 in JKT, $0.021 in NSUK and $0.017 in ABIIRT (in descending order of the induced income effects).

While the column totals in Table 2 indicate "the induced effects *originating* from each region, the values of the row totals ... show the induced effects *recieved* in each region due to expenditure from 1 unit of income in the regions of origin" (Miyazawa, 1976, p. 28, italics kept). In terms of column sums, the largest global household income multiplier of $2.488 is observed for USMCA, while the lowest multiplier of $1.565 for CHN. In terms of the row sums, the largest induced effects received per $1 earned income simultaneously in *all* of the regions are found for the USMCA and EU regions, which amount to $2.650 and $2.093, respectively. The NSUK region makes the lowest score in this respect by receiving only $1.729 induced effects. In line with the results of earlier studies (see e.g. Miyazawa, 1976; Hewings et al., 2001), we find that also at the global level the income multipliers showing induced effects originating from each region are relatively more homogeneous than the induced effects received in each region.[8]

[6] Here, the underlying 44×44 matrix **K**, which along with other detailed results are available as supplementary material to this essay, had to be aggregated to a 7×7 matrix. Row-wise aggregation is a simple summation across respective countries of each aggregate region. However, as the values down each column of **K** show the total induced income effects per one unit of initial induced income in the region under consideration, for column-wise aggregation we take the weighted average of the columns in the original **K** corresponding to each aggregate region, with weights representing the respective initial induced income, $\mathbf{W}_c\mathbf{Lf}^*$; this is consistent with the reasoning of Miyazawa's "fundamental equation of income formation", equation (9b) with $\mathbf{g} = \mathbf{0}$.

[7] We note that the income multipliers examined here account only for the impacts generated by *wage and salary income* due to endogenizing households' activity. Therefore, the model does not endogenously (i.e. with further multiplicative effects) consider other sources of income (Pyatt, 2001).

[8] These indicators obtained from the full WIOT show e.g. that the sample standard deviation of the row sums of the 44×44 interrelational income multiplier matrix **K** was found to be more than three times bigger than that of its column sums (0.870 vs. 0.287). This relation also holds true for their relative standard deviations (or coefficients

One might also be interested in the extent of household income *spillover* effects to or from other regions, i.e. focus instead on the column and row sums of only the *off-diagonal* elements in the interregional income multiplier matrix. The relevant totals, designated as "Total (other regions)" in Table 2, show the corresponding values of the total income-induced spillover effects. So the RoW region has by far the largest global income spillover effects originating from it: $1 household income earned in RoW generates overall $0.635 of earned income in the remaining six regions. ABIIRT and NSUK come next in the list, but have the corresponding spillover induced effects of a much smaller size of $0.352 and $0.327, respectively. On the other hand, $1 income earned in *other* regions generates total earned income of $0.499 in EU, $0.389 in RoW and $0.385 in USMCA.

All in all, if we consider RoW as the less advanced region (on average), then Miyazawa's conclusion of "concentration of income formation in the advanced area" also follows from our results discussed so far, i.e. "there is a ... tendency for induced income to flow from the backward areas to the advanced area(s)" (Miyazawa, 1976, p. 28).

Table 3: Asymmetry in the income-induced spillover effects, 2014 (%)

Region of income receipt	Region of income origin						
	ABIIRT	CHN	JKT	EU	NSUK	USMCA	RoW
ABIIRT	–	11.2	10.7	10.0	5.9	7.6	9.8
CHN	17.6	–	20.1	14.9	9.2	19.7	18.9
JKT	8.4	13.3	–	6.2	4.2	11.5	11.0
EU	22.8	19.9	14.5	–	46.4	24.0	25.0
NSUK	6.4	5.6	4.8	17.5	–	9.4	9.2
USMCA	15.1	17.9	18.0	24.0	17.1	–	26.0
RoW	29.8	32.1	32.0	27.4	17.2	27.9	–
Total spillover effects	100.0	100.0	100.0	100.0	100.0	100.0	100.0
Spillover-to-total multiplier	16.6	6.3	13.4	11.1	17.2	8.9	28.2

Note: "Spillover-to-total multiplier" indicates the relative size (in %) of the total spillover effects in the global income-induced multiplier (obtained by dividing the figures along the row "Total (other regions)" to those along "Total (all regions)" in Table 2).

To further highlight the asymmetry in the income-induced spillover effects, Table 3 presents the relative size of the off-diagonal elements in the interrelational-interregional income multiplier matrix (Table 2) as percentage of the corresponding total spillover effects. Consider as an example the "trading pair" EU vs. JKT, both of which have roughly similar relative size of total spillover effects in their income-induced global multipliers (of, respectively, 11.1% and 13.4%). Table 3 shows that 14.5% of JKT's total spillover effects ends up as earned incomes of the EU households, whereas only 6.2% of EU's global income spillover effects reaches JKT households. Also note that, for example, almost half - namely 46.4% - of NSUK's spillover effects benefit EU households, while the allocation of EU income spillover effects is much less concentrated across their receiving ends.

of variation) comparison, as the means of the two income multiplier indicators are equal by construction. In addition, the obtained correlation coefficient between these two indicators of only 0.253 confirms earlier empirical findings that the information given by these two income multipliers are empirically different.

Table 4: Total (initial, direct and indirect) income formation by regional demand, 2014

Region of income receipt	Region of final demand origin							Total (all)	Total (other)
	ABIIRT	CHN	JKT	EU	NSUK	USMCA	RoW		
(a) Total income formation due to regional autonomous final demand (bln. US$)									
ABIIRT	2,677	111	73	121	30	108	287	3,408	730
CHN	179	4,284	158	201	51	303	491	5,666	1,382
JKT	101	191	2,283	99	28	211	303	3,217	934
EU	264	228	123	5,410	275	374	694	7,368	1,959
NSUK	67	58	38	236	1,322	133	237	2,090	768
USMCA	167	192	142	352	118	8,853	648	10,472	1,619
RoW	291	334	219	322	85	366	3,290	4,907	1,617
Total (all)	3,747	5,398	3,036	6,741	1,910	10,347	5,950	37,128	
Total (other regions)	1,069	1,114	752	1,331	588	1,494	2,661		9,009
(b) Dependency of income formation by regional autonomous final demand (%)									
ABIIRT	78.6	3.3	2.1	3.6	0.9	3.2	8.4	100.0	21.4
CHN	3.2	75.6	2.8	3.5	0.9	5.3	8.7	100.0	24.4
JKT	3.1	5.9	71.0	3.1	0.9	6.5	9.4	100.0	29.0
EU	3.6	3.1	1.7	73.4	3.7	5.1	9.4	100.0	26.6
NSUK	3.2	2.8	1.8	11.3	63.3	6.3	11.3	100.0	36.7
USMCA	1.6	1.8	1.4	3.4	1.1	84.5	6.2	100.0	15.5
RoW	5.9	6.8	4.5	6.6	1.7	7.5	67.0	100.0	33.0
Total (all)	10.1	14.5	8.2	18.2	5.1	27.9	16.0	100.0	
(c) Contribution of regions of demand origin to income formation in regions of income receipt (%)									
ABIIRT	71.5	2.1	2.4	1.8	1.6	1.0	4.8	9.2	
CHN	4.8	79.4	5.2	3.0	2.7	2.9	8.3	15.3	
JKT	2.7	3.5	75.2	1.5	1.5	2.0	5.1	8.7	
EU	7.0	4.2	4.1	80.3	14.4	3.6	11.7	19.8	
NSUK	1.8	1.1	1.3	3.5	69.2	1.3	4.0	5.6	
USMCA	4.5	3.6	4.7	5.2	6.2	85.6	10.9	28.2	
RoW	7.8	6.2	7.2	4.8	4.5	3.5	55.3	13.2	
Total (all)	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	
Total (other regions)	28.5	20.6	24.8	19.7	30.8	14.4	44.7		

However, the interregional income multipliers do not account for the structure and differences in product composition of each region's exogenous (or, equivalently, autonomous) final demand, which excludes household consumption expenditures. The results of Miyazawa's "fundamental equation of income formation", where the total autonomous final demand is separated into each region's demand (for both domestically produced and imported products), are presented in Table 4.[9] Thus, *the rs-th element of the total income formation matrix represents the absolute value of total household income in region r generated due to the autonomous final expenditures originating from region s*. By construction, the total earned income of $37,128 bln – shown in part (a) of Table 4 – represents (the estimate of) total compensation of employees at the global level in 2014, which makes up 48.2% of the world GDP.

[9]Formally, the total income formation matrix, whose aggregated version is given in Table 4, is obtained from $\mathbf{Y} = \mathbf{KW}_c\mathbf{LF}^*$, where the columns of $\mathbf{F}^*$ represent total autonomous demands of each region. For further details, see the Appendix.

Table 4-(b) gives the percentage dependencies of income formation by regional demand (these figures are obtained by dividing the row elements of the income-formation matrix in (a) by the corresponding totals shown in its ninth column). For the world as a whole, 76.6% of all the earned income comes directly and indirectly from the initial expenditures in USMCA (27.9%), EU (18.2%), RoW (16.0%) and CHN (14.5%). The corresponding contributions of autonomous expenditures in ABIIRT, JKT and NSUK to global income formation are, respectively, 10.1%, 8.2% and 5.1%. These overall dependency indicators strongly resemble the relative size of GDP of the corresponding regions (see Figure 1).

A closer look into the row elements of individual regions in Table 4-(b) reveals that USMCA has the largest "self-dependent ratio of income formation". That is, 84.5% of income generated in USMCA comes directly and indirectly from initial expenditures originating in USMCA itself; thus, the USMCA income formation dependence on all other regions' demand was found to be only 15.5%, as also shown in the last column of Table 4-(b). On the other hand, NSUK is found to be the least self-dependent region: its self-dependency ratio is only 63.3%, while the region's income formation is largely dependent on demand originating, particularly, in the EU and RoW regions: both of these regions identically contribute 11.3% each to NSUK's household income generation.[10] Also note that the RoW region is the second region that is highly dependent on the outside-originating autonomous expenditures: 33.0% of income generated in RoW is due to the outside autonomous demand, particularly coming from USMCA (7.5%), CHN (6.8%), EU (6.6%) and ABIIRT (5.9%).

Table 4-(c) shows the relative size of income formation in all regions due to expenditures in each region of demand origin (i.e. these figures are obtained by dividing the column elements of the income-formation matrix in (a) by the corresponding totals shown along its 'Total (all)' row). Thus, we observe that expenditures in RoW globally generated $5,950 bln of earned income, but 44.7% of it is earned outside the RoW region. Similarly, a high "degree of leakage" of 30.8% is found for NSUK. In line with our earlier discussions, not surprisingly the lowest "leakage ratio" of 14.4% is found for the USMCA region.

Miyazawa's "multi-sector income multiplier" or "matrix multiplier of income formation" cannot be easily reported because of its large dimension, which in effect is a generalization of the Keynesian macro-multiplier to a multi-industry setting.[11] It seems that this was the reason why Miyazawa reported an aggregated version of this matrix, where the columns of the original matrix corresponding to each region are weighted averages, with weights representing exogenous final demand shares of the region in question. The obtained figures are referred to as "the coefficients of inducement to income per unit of autonomous demand by each region"

[10] In view of BREXIT, it is interesting to see the corresponding earned income inter-dependency ratios between the EU and the UK. The corresponding detailed results for 2014 indicate that 10.4% of household income generated in the UK is due to (autonomous) expenditures in the EU-27, whereas, on average, only 2.6% of income formation in 27 EU countries is due to (autonomous) demand originating in the UK. From this perspective, one might conclude that BREXIT is expected to reduce the UK household income formation (roughly four times) more than that in an average EU country. For further details, see the disaggregated results in the supplementary material.

[11] Formally, this multiplier matrix is obtained from $\mathbf{KW}_c\mathbf{L}$ (see the Appendix), and in our case has the dimension of 44×2464, i.e. (number of regions)$\times$(number of industries)(number of regions).

Table 5: Coefficients of income inducement per $1 of regional autonomous demand, 2014

Region of income receipt	Region of final demand origin							
	ABIIRT	CHN	JKT	EU	NSUK	USMCA	RoW	All regions
ABIIRT	0.707	0.018	0.026	0.019	0.018	0.015	0.053	0.102
CHN	0.047	0.693	0.057	0.032	0.030	0.042	0.090	0.169
JKT	0.027	0.031	0.827	0.016	0.017	0.029	0.056	0.096
EU	0.070	0.037	0.045	0.854	0.163	0.052	0.127	0.220
NSUK	0.018	0.009	0.014	0.037	0.785	0.018	0.043	0.062
USMCA	0.044	0.031	0.051	0.055	0.070	1.225	0.118	0.313
RoW	0.077	0.054	0.079	0.051	0.051	0.051	0.602	0.147
Total (all)	0.989	0.874	1.099	1.064	1.133	1.431	1.088	1.110
Total (other regions)	0.282	0.180	0.272	0.210	0.349	0.207	0.487	

Note: Autonomous final demand includes government consumption, gross capital formation and final exports.

(Miyazawa, 1976, p. 30). This income-inducement coefficients matrix in our empirical application case is presented in Table 5.[12] The column sums of these coefficients, i.e. figures along the row 'Total (all)', give *global household income multipliers per $1 of autonomous demand in each region of demand origin*. Miyazawa called these column sums as "the induced effects *by region of origin*", while the ratio of total income *received* by each region to global autonomous demand (i.e. the first seven entries of the last column in Table 5) as "the induced effects *by region of receipt*" (p. 30, italics kept).[13]

With endogenous household activities, the Miyazawa multi-sector income-multiplier or the income-inducement coefficients matrix indicates that *given the structure of production and final demand of the 2014 WIOT, a $1 increase in the world autonomous demand raises earned income globally, on average, by $1.110* (see row 'Total (all)' in the last column of Table 5). However, there are marked differences across the regions. For example, $1 increase in autonomous final demand in USMCA raises earned income globally, on average, by $1.431, whereas Chinese final expenditures' average global impact is only $0.874. Note that "the induced effects by region of origin" could be less than one because here only compensation of employees drive the income formation process, while all other components of GVA are kept outside the multiplicative demand-production-income interactions captured within the model (and neither the direct effects of these other components of GVA are included in the reported multipliers).

The "induced effects by region of receipt", similar to the corresponding summary indicator from the interregional income multipliers, confirm that the largest income receivers are the

[12] Note that alternatively the income-inducement coefficients can be obtained by dividing the column elements in the income formation matrix in Table 4-(a) by the corresponding regional autonomous final demand totals.

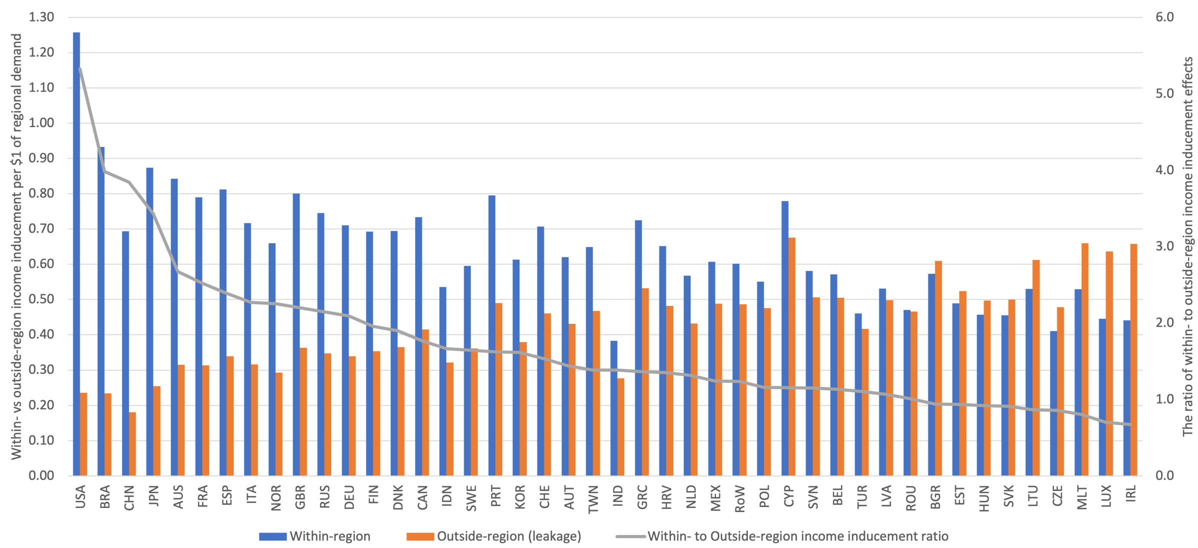
[13] If we denote the elements of the income-formation matrix in Table 4-(a) by y_{ij}, then the "induced effects by region of receipt" of region i can be found as

$$\frac{\sum_j y_{ij}}{\sum_k f_k} = \frac{\sum_j y_{ij}}{f_j}\frac{f_j}{\sum_k f_k} = \sum_j \frac{y_{ij}}{f_j}\omega_j,$$

where f_j is the total autonomous final demand in region j and $\omega_j \equiv f_j / \sum_k f_k$ is its corresponding demand share (weight) in the nation (or world) autonomous demand. That is why Miyazawa uses "Average" when referring to this latter summary indicator.

USMCA and EU regions, while the least income beneficiaries are JKT and NSUK. That is, *given the production and demand structure of the 2014 WIOT, a $1 increase in worldwide (autonomous) demand generates, on average, $0.313, $0.220, $0.169, $0.102, $0.096 and $0.062 of earned income, respectively, in USMCA, EU, CHN, ABIIRT, JKT and NSUK.* Note that in contrast to the interrelational income multipliers, according to which CHN was the fifth largest receiver of income, now according to the induced effects by region of receipt CHN takes the third top position. Similarly, now RoW loses its standing compared to what was implied by Table 2. Another important difference between the induced effects received in each region (Table 2) and the average induced effects by region (Table 5) is that the relative differences of regions according to these two summary indicators are much larger in the latter case. This confirms similar results found in the literate (e.g. Miyazawa, 1976; Okuyama et al., 1999; Hewings et al., 2001) for the global economy as well. Such differences between the two income multipliers imply that "the location of autonomous demand has a substantial effect in determining regional income generation, especially in the income-receiving base" (Miyazawa, 1976, p. 30). Finally, in terms of specific intra- and inter-regional details, the results in Table 5 largely convey similar message as those reported in Table 4-(c) due to their roughly similar derivation. Thus, we do not further examine the details of regional self-dependency and leakage extent of income formation.

Figure 2: Within- vs. outside-country income inducement per $1 of country's autonomous final demand, 2014



Note: For country abbreviations, see the supplementary material (Excel file). Here, "region" refers to "country". *Source*: Own elaborations based on WIOD Release 2016.

To give a flavor of the results at the WIOD country-level detail, Figure 2 shows the decomposition of total household income inducement per $1 of regional autonomous final demand by country (region) of demand origin into its within- and outside-country components. Clearly, the US stands out as the most "self-dependent country" in its income formation process, while small open economies such as Ireland, Luxembourg and Malta show the highest degree of income inducement leakages. Note that each country unitary (autonomous) final demand vector includes not only (the shares of) domestically produced goods but also foreign final products. This explains why for some countries the outside-country income inducement effects could be (ultimately) larger than their within-country income impacts.

As a final note, we have also calculated the considered two (aggregate) multiplier matrices for the year of 2008, and compared them with the counterpart matrices reported in Tables 2 and 5. It was found that, compared with 2008, all the elements of the interrelational income multiplier matrix in 2014 *decreased*, on average, by about 86%. The corresponding figure for the income-inducement coefficients matrix was found to be of a similar magnitude, i.e. -85%. The seven *intra*-regional elements in these matrices, however, were found to have *increased*, on average, by 0.8% and 7.7%, respectively. These results partly reflect the consequences of the 2008 global financial crisis when international trade dropped much faster than the world GDP (see e.g. Bems et al., 2011).[14] The details of this exercise are not further presented and discussed here, as this falls outside the scope of the current essay. A complete analysis of changes of income multiplier matrices requires an explanation of the drivers of their evolution over time, which could be done using the techniques of structural decomposition analysis.

[14] Related results are obtained in Temursho (2018), which finds that the global and interregional feedback-spillover output multipliers had an average upward trend since 1995 and achieved their peak in 2008, while thereafter all started to decline in 2009 (which is the final year of the time period covered in the study).

Appendix: A detailed account of Miyazawa's income multipliers

Without loss of generality, consider an hypothetical world WIOT setting with three countries, each with n industries. Then the $3n \times 3n$ matrix of intermediate interindustry transactions, the $3n \times 3$ matrix of final demands, the $3n$-dimensional vector of gross value added (GVA) plus taxes less subsidies on products (TLS) for intermediate use, and the $3n$-dimensional vector of gross outputs have the following respective forms:[15]

$$\mathbf{Z} = \begin{bmatrix} \mathbf{Z}^{11} & \mathbf{Z}^{12} & \mathbf{Z}^{13} \\ \mathbf{Z}^{21} & \mathbf{Z}^{22} & \mathbf{Z}^{23} \\ \mathbf{Z}^{31} & \mathbf{Z}^{32} & \mathbf{Z}^{33} \end{bmatrix}, \quad \mathbf{F} = \begin{bmatrix} \mathbf{f}^{11} & \mathbf{f}^{12} & \mathbf{f}^{13} \\ \mathbf{f}^{21} & \mathbf{f}^{22} & \mathbf{f}^{23} \\ \mathbf{f}^{31} & \mathbf{f}^{32} & \mathbf{f}^{33} \end{bmatrix}, \quad \mathbf{v} = \begin{bmatrix} \mathbf{v}^{1} \\ \mathbf{v}^{2} \\ \mathbf{v}^{3} \end{bmatrix} \text{ and } \mathbf{x} = \begin{bmatrix} \mathbf{x}^{1} \\ \mathbf{x}^{2} \\ \mathbf{x}^{3} \end{bmatrix},$$

where e.g. $\mathbf{Z}^{rs}$ is the $n \times n$ matrix of intermediate deliveries from industries in country r to industries in country s.

The output-side IO accounting identity states that the output supply of each industry is equal to the sum of intermediate and final demands for the products produced by the industry in question, i.e.

$$\mathbf{x} = \mathbf{Z}\boldsymbol{\imath} + \mathbf{F}\boldsymbol{\imath}, \tag{1}$$

where $\boldsymbol{\imath}$ is the summation vector of ones. Now define the *input coefficients* as intermediate deliveries per unit of output of each purchasing industry, i.e. $a_{ij}^{rs} = z_{ij}^{rs}/x_j^s$, or in compact matrix notation, $\mathbf{A} = \mathbf{Z}\hat{\mathbf{x}}^{-1}$. Plugging this latter expression into (1) gives

$$\mathbf{x} = \mathbf{A}\mathbf{x} + \mathbf{f}, \tag{2}$$

where $\mathbf{f} \equiv \mathbf{F}\boldsymbol{\imath}$. Solving (2) for the vector of outputs yields the solution of the basic open demand-driven IO quantity model (Leontief, 1936, 1986):

$$\mathbf{x} = \mathbf{L}\mathbf{f}, \tag{3}$$

where $\mathbf{L} \equiv (\mathbf{I} - \mathbf{A})^{-1}$ is the well-known *Leontief inverse* matrix, whose typical element l_{ij}^{rs} indicates the (extra) output of industry i in country r that is directly and indirectly required to satisfy one (extra) unit of final demand for products of industry j in country s.

The core assumptions of the model are fixed input coefficients and fixed product prices. As such this model is useful for short-term impact analysis purposes, and is particularly relevant when the economy (or economies) under consideration are under-employing factors of production such as labor and capital.

Any other policy-relevant impacts (e.g. income, employment, emissions) can be found by linking the variable(s) of interest to gross outputs of industries. For example, if we denote the *direct income coefficients* (or value-added to output ratios) vector by $\mathbf{v}_c = \hat{\mathbf{x}}^{-1}\mathbf{v}$, then income generated due to an exogenously specified final demand shock $\mathbf{f}$ can be straightforwardly derived

[15] Matrices are given in bold, capitals; vectors in bold, lower cases; and scalars in italicized, lower case letters. Vectors are columns by definition, while row vectors are obtained by transposition, indicated by a prime. $\hat{\mathbf{x}}$ denotes a diagonal matrix with the entries of $\mathbf{x}$ on its main diagonal and zeros elsewhere.

from the following $3n \times 3n$ matrix:

$$\widetilde{\mathbf{V}} = \hat{\mathbf{v}}_c \mathbf{L}\hat{\mathbf{f}}. \tag{4}$$

The typical element $\tilde{v}_{ij}^{rs}$ of $\widetilde{\mathbf{V}}$ in (4) indicates the total income generated in the production process of industry i in country r that is directly and indirectly necessary to satisfy final demand for products of industry j in country s, f_j^s. Depending on the research question, various indicators based on different aggregation/summation of the elements of $\widetilde{\mathbf{V}}$ can be used.

In (3) and (4), final demand categories (private consumption, government consumption, gross capital formation, and exports) were assumed to be exogenous. An extension of the basic IO model thus calls for endogenization of a part of final demand. One such widely used *semi-closed* IO model treats households similar to industries on the grounds that: (a) households earn income in exchange for their labor inputs to production, and (b) as consumers, households spend income on products in "rather well patterned ways" (Miller and Blair, 2009, p. 35). Households as a choice of final demand endogenization is additionally driven by the fact that private consumption accounts for a significant part (about 60%) of GDP in most (developed) countries.

Within a single-country IO framework, it is particularly important to take account of household heterogeneity, e.g. in terms of distinct income groups. Such an IO setting was first examined by Miyazawa and Masegi (1963) and Miyazawa (1968). The latter work, which is focused on interregional income distribution in Japan, is directly applicable for our purposes of quantifying global income multipliers.

In our three-country IO setting, Miyazawa's IO system, which is an "augmented" version of (2), can be formulated as follows:

$$\begin{bmatrix} \mathbf{x}^1 \\ \mathbf{x}^2 \\ \mathbf{x}^3 \\ \hline y^1 \\ y^2 \\ y^3 \end{bmatrix} = \left[\begin{array}{ccc|ccc} \mathbf{A}^{11} & \mathbf{A}^{12} & \mathbf{A}^{13} & \mathbf{h}_c^{11} & \mathbf{h}_c^{12} & \mathbf{h}_c^{13} \\ \mathbf{A}^{21} & \mathbf{A}^{22} & \mathbf{A}^{23} & \mathbf{h}_c^{21} & \mathbf{h}_c^{22} & \mathbf{h}_c^{23} \\ \mathbf{A}^{31} & \mathbf{A}^{32} & \mathbf{A}^{33} & \mathbf{h}_c^{31} & \mathbf{h}_c^{32} & \mathbf{h}_c^{33} \\ \hline (\mathbf{w}_c^{11})' & (\mathbf{w}_c^{12})' & (\mathbf{w}_c^{13})' & 0 & 0 & 0 \\ (\mathbf{w}_c^{21})' & (\mathbf{w}_c^{22})' & (\mathbf{w}_c^{23})' & 0 & 0 & 0 \\ (\mathbf{w}_c^{31})' & (\mathbf{w}_c^{32})' & (\mathbf{w}_c^{33})' & 0 & 0 & 0 \end{array}\right] \begin{bmatrix} \mathbf{x}^1 \\ \mathbf{x}^2 \\ \mathbf{x}^3 \\ \hline y^1 \\ y^2 \\ y^3 \end{bmatrix} + \begin{bmatrix} (\mathbf{f}^*)^{1\bullet} \\ (\mathbf{f}^*)^{2\bullet} \\ (\mathbf{f}^*)^{3\bullet} \\ \hline g^1 \\ g^2 \\ g^3 \end{bmatrix}, \tag{5}$$

where the following new variables are introduced:

- y^r is total earned income of households that are residents in country r,
- g^r is total exogenous income of households in country r, which includes such items as social security benefits, pensions, incomes from financial assets and from the rest of the world,
- $\mathbf{w}_c^{rs}$ is the vector of *households input coefficients*, whose jth element indicates households' income in country r earned from one unit of production (output) of industry j in country s, i.e. $w_{cj}^{rs} = w_j^{rs}/x_j^s$,
- $\mathbf{h}_c^{rs}$ is the vector of *consumption coefficients*, whose ith entry gives consumption expenditure of households in country s for products of industry i produced in country r from (per) income earned in country s, i.e. $h_{ci}^{rs} = h_i^{rs}/y^s$, and

- $(\mathbf{f}^*)^{r\bullet}$ – is the remaining total final demand provided by industries from country r that excludes households' consumption expenditures, i.e. $(\mathbf{f}^*)^{r\bullet} \equiv \sum_s (\mathbf{f}^*)^{rs} = \sum_s (\mathbf{f}^{rs} - \mathbf{h}^{rs})$.

In our empirical application, we use compensation of employees for earned incomes $\mathbf{w}^{rs}$ and final consumption expenditure by households for $\mathbf{h}^{rs}$. However, compared to the latter data, in currently available global IOTs, data on $\mathbf{w}^{rs}$ (or generally on $\mathbf{v}^{rs}$) with $r \neq s$ are missing. The empirical importance of distinguishing value added components by the country of origin of primary inputs' providers and countries of destination where this value added is created is stressed elsewhere (see e.g. Kanemoto et al., 2012; Temursho and Miller, 2020). Thus, in the empirical application of Miyazawa's approach within the global inter-country IO framework, we end up with setting $\mathbf{w}^{rs} = \mathbf{0}$ for all $r \neq s$.

For simplicity of exposition of the solution of Miyazawa approach, let us further denote:

$$\mathbf{H}_c = \begin{bmatrix} \mathbf{h}_c^{11} & \mathbf{h}_c^{12} & \mathbf{h}_c^{13} \\ \mathbf{h}_c^{21} & \mathbf{h}_c^{22} & \mathbf{h}_c^{23} \\ \mathbf{h}_c^{31} & \mathbf{h}_c^{32} & \mathbf{h}_c^{33} \end{bmatrix}, \quad \mathbf{W}_c = \begin{bmatrix} (\mathbf{w}_c^{11})' & (\mathbf{w}_c^{12})' & (\mathbf{w}_c^{13})' \\ (\mathbf{w}_c^{21})' & (\mathbf{w}_c^{22})' & (\mathbf{w}_c^{23})' \\ (\mathbf{w}_c^{31})' & (\mathbf{w}_c^{32})' & (\mathbf{w}_c^{33})' \end{bmatrix}, \quad \mathbf{f}^* = \begin{bmatrix} (\mathbf{f}^*)^{1\bullet} \\ (\mathbf{f}^*)^{2\bullet} \\ (\mathbf{f}^*)^{3\bullet} \end{bmatrix}, \quad \mathbf{y} = \begin{bmatrix} y^1 \\ y^2 \\ y^3 \end{bmatrix} \text{ and } \mathbf{g} = \begin{bmatrix} g^1 \\ g^2 \\ g^3 \end{bmatrix}.$$

Then the system (5) can be alternatively written as

$$\mathbf{x} = \mathbf{A}\mathbf{x} + \mathbf{H}_c\mathbf{y} + \mathbf{f}^*, \tag{6a}$$

$$\mathbf{y} = \mathbf{W}_c\mathbf{x} + \mathbf{g}. \tag{6b}$$

Equations (6a)-(6b) imply $\mathbf{x} = \mathbf{A}\mathbf{x} + \mathbf{H}_c\mathbf{W}_c\mathbf{x} + \mathbf{H}_c\mathbf{g} + \mathbf{f}^*$, which makes explicit Miyazawa's "disaggregated consumption function" as:

$$\mathbf{f}_h = \mathbf{H}_c\mathbf{W}_c\mathbf{x} + \mathbf{H}_c\mathbf{g}. \tag{7}$$

This function includes both consumption from endogeous income earned in the production process, $\mathbf{H}_c\mathbf{W}_c\mathbf{x}$, and consumption from exogeonous income, $\mathbf{H}_c\mathbf{g}$. In the most disaggregated form, for example, the typical element of the $3n \times 3n$ matrix $\mathbf{H}_c\mathbf{W}_c\hat{\mathbf{x}}$, which underlies the first term in (7), i.e. $(\mathbf{H}_c\mathbf{W}_c\hat{\mathbf{x}})_{ij}^{rs}$, indicates all households' (extra) consumption of products produced by industry i in country r from their (extra) earned income in industry j in country s. "If we add nonhomogenous terms, or exogenous elements to the consumption function, $[\mathbf{H}_c]$ becomes the matrix of *marginal coefficients*, and in this case we can include the nonhomogenous terms in $[\mathbf{f}^*]$" (Miyazawa, 1976, p. 5, italics added).

Solving (6a)-(6b) for $\mathbf{x}$ is straightforward and yields one formulation of Miyazawa system's solution as follows:

$$\mathbf{x} = \tilde{\mathbf{L}}\mathbf{f}^* + \tilde{\mathbf{L}}\mathbf{H}_c\mathbf{g}, \tag{8a}$$

$$\mathbf{y} = \mathbf{W}_c\tilde{\mathbf{L}}\mathbf{f}^* + (\mathbf{I} + \mathbf{W}_c\tilde{\mathbf{L}}\mathbf{H}_c)\mathbf{g}, \tag{8b}$$

where $\tilde{\mathbf{L}} \equiv (\mathbf{I} - \mathbf{A} - \mathbf{H}_c\mathbf{W}_c)^{-1}$ is "the *enlarged inverse matrix multiplier* showing the total effects of exogenous final demand on outputs via interindustry and induced consumption activities"

(Miyazawa, 1976, p. 5). Note that one can write $\mathbf{H}_c\mathbf{W}_c = \mathbf{h}_c^{\bullet 1}\mathbf{w}_c^{1\bullet} + \mathbf{h}_c^{\bullet 2}\mathbf{w}_c^{2\bullet} + \cdots + \mathbf{h}_c^{\bullet n}\mathbf{w}_c^{n\bullet}$, where $\mathbf{h}_c^{\bullet r}$ is the rth column of $\mathbf{H}_c$ and $\mathbf{w}_c^{r\bullet}$ is the rth row of $\mathbf{W}_c$.[16] Hence, the standard semi-closed IO model with endogenized aggregate households of only one type and with $\mathbf{g} = \mathbf{0}$ (Miller and Blair, 2009, Ch. 2.5) is a particular case of Miyazawa's model.

The second term in (8a), $\tilde{\mathbf{L}}\mathbf{H}_c\mathbf{g}$, quantifies total outputs necessary to satisfy additional consumption demand from households' exogenous income $\mathbf{H}_c\mathbf{g}$. Thus, the total earned income of households, $\mathbf{y}$, apart from exogenous income g, also includes all incomes generated in the production process that fully accounts for the corresponding intermediate and induced consumption demands (which explains the presence of $\tilde{\mathbf{L}}$ in the given expressions) in order to satisfy: (a) exogenous final demand, $\mathbf{W}_c\tilde{\mathbf{L}}\mathbf{f}^*$, and (b) the additional consumption demand arising from exogenous income, $\mathbf{W}_c\tilde{\mathbf{L}}\mathbf{H}_c\mathbf{g}$. All these income categories are captured in (8b).

However, there is an alternative to (8a)-(8b) formulation of Miyazawa system's solution, but exactly equivalent in terms of final results, that is used (more often) in empirical applications for the reasons that become clear soon. Using the results on the inverse of partitioned matrices (see e.g. Abadir and Magnus, 2005, p. 106), the solution of Miyazawa system (5) can be alternatively written as:

$$\mathbf{x} = \mathbf{L}(\mathbf{I} + \mathbf{H}_c\mathbf{K}\mathbf{W}_c\mathbf{L})\mathbf{f}^* + \mathbf{L}\mathbf{H}_c\mathbf{K}\mathbf{g}, \tag{9a}$$

$$\mathbf{y} = \mathbf{K}\mathbf{W}_c\mathbf{L}\mathbf{f}^* + \mathbf{K}\mathbf{g}, \tag{9b}$$

where $\mathbf{K} \equiv (\mathbf{I} - \mathbf{W}_c\mathbf{L}\mathbf{H}_c)^{-1}$ is referred to as the "*interrelational regional income multiplier matrix*" (Miyazawa, 1976, p. 25), which in the considered global IO setting could be equally termed as the "*inter-country* (or *interregional*) *income multiplier matrix*". To understand the reasoning of such terminology, note that the typical element of the 3×3 matrix $\mathbf{W}_c\mathbf{L}\mathbf{H}_c$, i.e. $(\mathbf{W}_c\mathbf{L}\mathbf{H}_c)^{rs} = \sum_j \sum_k (\mathbf{w}_c^{rj})'\mathbf{L}^{jk}\mathbf{h}_c^{ks}$, shows the direct increase in earned income in region r generated from the expenditure of one unit of additional income in region s. The term "direct" here refers to the first cycle in the expenditure-production-income rounds, and not to the standard total output-intermediate demand rounds. In what follows, and also in our application, it is assumed that exogenous incomes do not change and thus one could set $\mathbf{g} = \mathbf{0}$.

In Table 6 we look closer into the output, income and consumption effects of a final demand stimulus $\mathbf{f}^*$ at each step of the income-formation process or the "successive income generating process" (Miyazawa, 1976, p. 25). Within the open IO framework, the output and income effects would amount to $\mathbf{L}\mathbf{f}^*$ and $\mathbf{W}_c\mathbf{L}\mathbf{f}^*$, respectively. These terms show up, respectively, in the second and third columns and both along the second row of Table 6. Whilst these are "final or total effects" within the open IO setting that accounts for all production rounds of total output-intermediate demand interactions, within the considered semi-closed IO model they represent only the "initial effects" in the income-formation process due to endogenization of households activities. Hence, the corresponding income-formation round along the second row of Table 6 is set to zero. In addition, now consumers do react to their initial increased in-

[16] For simplicity, we do not add transposition to $\mathbf{w}_c^{r\bullet}$ since the dot symbol as its second superscript should make it clear that the corresponding vector is a row vector, i.e $\mathbf{w}_c^{r\bullet} = [(\mathbf{w}_c^{r1})' \ (\mathbf{w}_c^{r2})' \ (\mathbf{w}_c^{r3})']$.

Table 6: Output, income and consumption effects of the income-formation process

Income formation rounds	Output effects (Output increases, $rn \times 1$ vector)	Income effects (Income increases, $r \times 1$ vector)	Consumption effects (Consumption increases, $rn \times 1$ vector)
0	$\mathbf{Lf}^*$	$\mathbf{W}_c\mathbf{Lf}^*$	$\mathbf{H}_c\mathbf{W}_c\mathbf{Lf}^*$
1	$\mathbf{LH}_c\mathbf{W}_c\mathbf{Lf}^*$	$(\mathbf{W}_c\mathbf{LH}_c)\mathbf{W}_c\mathbf{Lf}^*$	$\mathbf{H}_c(\mathbf{W}_c\mathbf{LH}_c)\mathbf{W}_c\mathbf{Lf}^*$
2	$\mathbf{LH}_c(\mathbf{W}_c\mathbf{LH}_c)\mathbf{W}_c\mathbf{Lf}^*$	$(\mathbf{W}_c\mathbf{LH}_c)^2\mathbf{W}_c\mathbf{Lf}^*$	$\mathbf{H}_c(\mathbf{W}_c\mathbf{LH}_c)^2\mathbf{W}_c\mathbf{Lf}^*$
3	$\mathbf{LH}_c(\mathbf{W}_c\mathbf{LH}_c)^2\mathbf{W}_c\mathbf{Lf}^*$	$(\mathbf{W}_c\mathbf{LH}_c)^3\mathbf{W}_c\mathbf{Lf}^*$	$\mathbf{H}_c(\mathbf{W}_c\mathbf{LH}_c)^3\mathbf{W}_c\mathbf{Lf}^*$
$\vdots$	$\vdots$	$\vdots$	$\vdots$
s	$\mathbf{LH}_c(\mathbf{W}_c\mathbf{LH}_c)^{s-2}\mathbf{W}_c\mathbf{Lf}^*$	$(\mathbf{W}_c\mathbf{LH}_c)^{s-1}\mathbf{W}_c\mathbf{Lf}^*$	$\mathbf{H}_c(\mathbf{W}_c\mathbf{LH}_c)^{s-1}\mathbf{W}_c\mathbf{Lf}^*$
$\vdots$	$\vdots$	$\vdots$	$\vdots$
Total	$\mathbf{Lf}^* + \mathbf{LH}_c\mathbf{KW}_c\mathbf{Lf}^*$	$\mathbf{KW}_c\mathbf{Lf}^*$	$\mathbf{H}_c\mathbf{KW}_c\mathbf{Lf}^*$

Note: r and n indicate, respectively, the number of regions (countries) and number of industries in each region. With households distinguished by income groups, r would refer to the number of distinct income groups.

come that results in the initial induced consumption increase of $\mathbf{H}_c\mathbf{W}_c\mathbf{Lf}^*$, which is shown in the second row and the fourth column of Table 6.

The increased initial households demand $\mathbf{H}_c\mathbf{W}_c\mathbf{Lf}^*$ needs to be satisfied, which if accounting for all direct and indirect production rounds within the standard open IO system, results in additional outputs amounting to $\mathbf{LH}_c\mathbf{W}_c\mathbf{Lf}^*$. Hence, pre-multiplication of the latter expression by the matrix of households inputs coefficients $\mathbf{W}_c$ and the resulting expression by the matrix of consumption coefficients $\mathbf{H}_c$ gives the corresponding income and consumption increases, respectively, as $(\mathbf{W}_c\mathbf{LH}_c)\mathbf{W}_c\mathbf{Lf}^*$ and $\mathbf{H}_c(\mathbf{W}_c\mathbf{LH}_c)\mathbf{W}_c\mathbf{Lf}^*$. These latter expressions are the direct effects in the income-formation process that account for the production-income-consumption interactions and thus appear along the third row of Table 6 that correspond to the 1st income-formation round. If we continue such recording of the sequences of the increased production-income-consumption effects – all due to the original final demand stimulus – along the entire income formation process, we end up with the respective expressions compactly shown in Table 6.

Looking down the third column (income effects) of Table 6, we can obtain the total income effects, namely, the initial, direct and indirect income effects along the entire income-formation process as follows:[17]

$$\begin{aligned} \mathbf{y} &= \left(\mathbf{I} + \mathbf{W}_c\mathbf{LH}_c + (\mathbf{W}_c\mathbf{LH}_c)^2 + (\mathbf{W}_c\mathbf{LH}_c)^3 + \cdots\right)\mathbf{W}_c\mathbf{Lf}^* \\ &= (\mathbf{I} - \mathbf{W}_c\mathbf{LH}_c)^{-1}\mathbf{W}_c\mathbf{Lf}^* = \mathbf{KW}_c\mathbf{Lf}^*, \end{aligned} \tag{10}$$

[17] For the technical details on the existence of related "enlarged inverse multiplier" matrices and the convergence of the power series in or similar to (10), the reader is referred to e.g. Takayama (1985); Miyazawa (1976) and Kimura and Kondo (1999).

which is exactly equivalent to (9b) with $\mathbf{g} = \mathbf{0}$. Explicitly considering also the standard total output-intermediate demand interactions results in an alternative decomposition of the income effects in (10) as:

$$\mathbf{y} = \underbrace{\mathbf{W}_c\mathbf{L}\mathbf{f}^*}_{\textit{initial, direct and indirect effects}} + \underbrace{(\mathbf{K} - \mathbf{I})\mathbf{W}_c\mathbf{L}\mathbf{f}^*}_{\textit{induced effects}}. \tag{11}$$

Note that (11) can be equally derived by pre-multiplying (9a) by $\mathbf{W}_c$, with zero exogenous income, $\mathbf{g} = \mathbf{0}$. Thus, (11) gives the standard initial, direct and indirect income effects that would have been obtained within an open IO system setting, plus the induced effects representing the additional increases in endogenous income due to closing the IO model with respect to (different types of) households.

Often the interrelational income multiplier matrix $\mathbf{K}$ is considered as a generalization of the Keynesian macro-multiplier to a multi-industry setting. Within the inter-country framework, the typical *rs*-th entry of $\mathbf{K}$ "represents the total household income of the *r*th region induced by expenditure from 1 unit of income earned in the *s*th region" (Miyazawa, 1976, p. 27).

However, if we want to express these latter interregional (or inter-income-group) income effects per unit of *autonomous* final demand (which is what the Keynesian multiplier represents at the aggregate macro level), then as is evident from (9b) the corresponding income multiplier matrix has to be $\mathbf{K}\mathbf{W}_c\mathbf{L}$. Miyazawa called this latter matrix as the "*multi-sector income multiplier*" or the "*matrix multiplier of income formation*". It is thus the explicit presence of the interrelational income multiplier and the multiplier of income formation, with their underlying usefull and policy-relevant information, that make the second formulation of Miyazawa system solution in (9a)-(9b) attractive. These two income multiplier matrices, as applied at the global, inter-country setting, are the main focus of this essay.

As a final note, notice that one could have equally summed the sequences of output, income and consumption effects, presented in Table 6, using an alternative power series expansion around $\mathbf{H}_c\mathbf{W}_c\mathbf{L}$ or $\mathbf{L}\mathbf{H}_c\mathbf{W}_c$ (instead of $\mathbf{W}_c\mathbf{L}\mathbf{H}_c$). This would have lead to alternative, but exactly identical in terms of final values, expressions. These alternative approaches "can be interpreted [respectively] as the propagation process viewed from [...] the consumption expenditure side [and] production side" (Miyazawa, 1976, p. 13). We let the reader her/him-self to discover these additional wonders of IO analysis.

References

Abadir, K. M. and J. R. Magnus: 2005, *Matrix Algebra*. Cambridge: Cambridge University Press.

Bems, R., R. C. Johnson, and K.-M. Yi: 2011, 'Vertical linkages and the collapse of global trade'. *American Economic Review: Papers and Proceedings* **101**, 308–312.

Dietzenbacher, E., B. Los, M. Timmer, and G. de Vries: 2013, 'The construction of world input-output tables in the WIOD project'. *Economic Systems Research* **25**(1), 71–98.

Hewings, G. J. D., Y. Okuyama, and M. Sonis: 2001, 'Economic interdependence within the Chicago metropolitan area: A Miyazawa analysis'. *Journal of Regional Science* **41**, 195–217.

Hewings, G. J. D., M. Sonis, M. Madden, and Y. Kimura: 1999, 'Introduction'. In: G. J. D. Hewings, M. Sonis, M. Madden, and Y. Kimura (eds.): *Understanding and Interpreting Economic Structure*. Berlin: Springer-Verlag, pp. 1–12.

Kanemoto, K., M. Lenzen, G. P. Peters, D. D. Moran, and A. Geschke: 2012, 'Frameworks for comparing emissions associated with production, consumption, and international trade'. *Environmental Science and Technology* **46**, 172–179.

Kimura, Y. and H. Kondo: 1999, 'Revisiting Miyazawa on multisectoral multipliers'. In: G. J. D. Hewings, M. Sonis, M. Madden, and Y. Kimura (eds.): *Understanding and Interpreting Economic Structure*. Berlin: Springer-Verlag, pp. 53–71.

Leontief, W. W.: 1936, 'Quantitative input-output relations in the economic system of the United States'. *Review of Economics and Statistics* **18**, 105–125.

Leontief, W. W.: 1986, *Input-Output Economics*. Oxford: Oxford University Press, 2 edition.

Miller, R. E. and P. D. Blair: 2009, *Input-Output Analysis: Foundations and Extensions*. Cambridge: Cambridge University Press, 2nd edition.

Miyazawa, K.: 1968, 'Input-output analysis and interrelational income multiplier as a matrix'. *Hitotsubashi Journal of Economics* **8**, 39–58.

Miyazawa, K.: 1976, *Input-Output Analysis and the Structure of Income Distribution*. Berlin: Springer-Verlag.

Miyazawa, K. and S. Masegi: 1963, 'Interindustry analysis and the structure of income-distribution'. *Metroeconomica* **15**, 89–103.

Okuyama, Y., M. Sonis, and G. J. D. Hewings: 1999, 'Economic impacts of an unscheduled, disruptive event: A Miyazawa multiplier analysis'. pp. 113–143.

Pyatt, G.: 2001, 'Some early multiplier models of the relationship between income distribution and production structure'. *Economic Systems Research* **13**(2), 139 – 163.

Streicher, G. and R. Stehrer: 2015, 'Whither Panama? Constructing a consistent and balanced world SUT system including international trade and transport margins'. *Economic Systems Research* **27**, 213–237.

Takayama, A.: 1985, *Mathematical Economics*. Cambridge: Cambridge University Press.

Temursho, U.: 2018, 'Intercountry feedback and spillover effects within the international supply and use framework: a Bayesian perspective'. *Economic Systems Research* **30**, 337–358.

Temursho, U. and R. E. Miller: 2020, 'Distance-based shared responsibility'. *Journal of Cleaner Production* **257**, 120481.

Timmer, M. P., E. Dietzenbacher, B. Los, R. Stehrer, and G. J. de Vries: 2015, 'An illustrated user guide to the World Input-Output Database: the case of global automotive production'. *Review of International Economics* **23**, 575–605.

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